

Bansho keikaku - planned board

1. Before the lesson: intent and planned final board

Type: launch, reactivation and conjecture

Kadai	“Equal angles $\Rightarrow$ proportional sides?”	Objective	Reactivate the congruence criteria (Year 9); discover that two triangles can have the same shape without having the same size, and conjecture that their sides are proportional. Skills: problem-solving, calculating.
Materials	Course notes booklet + exercise sheet + “Gammes du soir” (evening drills) sheet handed out; graduated ruler; the table from Section 3 of this guide in front of you.	Responses	P1 P2 P3 P4 anticipated; P4 is contingent if it brings in a new relationship.
ESS essential function CONT contingent form DECL response to prompt. Actual pupil responses are labelled C1, C2, C3 with no name or initial; C is then linked to the P functions.			

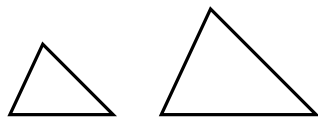
Planned final board at the end of the lesson.

IDEAL BOARD (final state): to project or reproduce

LEFT: the activity

Kadai: “Equal angles $\Rightarrow$ proportional sides?”

Two triangles with the same angles $65^\circ/45^\circ/70^\circ$, different sizes:



65/45/70

Measure the 6 sides, compare those opposite equal angles.

CENTRE: pupils' work

At random ($AB/B'C'$): ratios $\neq$ ✗ (not matched)

Opposite equal angles:

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC} \approx 1,5$$

Conjecture (pupils): “angles equal in pairs $\Rightarrow$ proportional sides”

But three measurements prove nothing: to be proved tomorrow.

RIGHT: definition (written summary)

Similar = proportional sides.

k : scale factor.

Corresponding sides / vertices = matching ones (opposite equal angles).

Tracks: M core / R recommended / H going further.

Physical board rule: problem, selected responses, refuted errors, connections and matome remain visible until the final photograph. Only auxiliary calculations may be erased after validation and verbalisation.

2. Storyboard: write, show, connect, keep, erase

Consistent colour code: main writing in the base colour; relationship, logical connector or invariant in an accent colour; error crossed out but still legible. Outside the auxiliary strip, no selected response is erased during the lesson.

Time	Board state	What is written	Talk and activity	Board action / status
Before entry	T0 - discreet outline	Draw three zones; reserve P1-P3 and the right-hand column.	Prepare the seating plan; do not pre-write the answer.	RESERVE spaces; everything remains open.
0–10	T1 - Phase 1 (0–10): Test 01 feedback, genuinely integrated	Project a real anonymised error; have pupils rephrase, correct and generalise it. Open the corresponding entry in the error log and announce the main error codes.	“On which line does the reasoning stop being justified, and what rule fixes it?”	KEEP the error, its correction and the general rule; put scripts away at 10 min.
10–15	T2 - Phase 2 (10–15): Flash quiz carried over from chapter 01	Give two questions with the same structure as the chapter 01 exercise sheet, exercise 5.12, with changed numbers. Collect at 15 min.	“Which move from chapter 01 do you need to spot before calculating?”	ERASE calculations after collection; keep only the reactivated move.
15–26	T3 - Phase 3 (15–26): Reading agreement and targeted diagnostic	Read the “Starting point” box, then complete the ten fluency skills in 8 min. Check items 5, 7 and 10 together; display the remaining answers.	“In this booklet, what will be required in the test, and what is there for mathematical culture?”	ERASE diagnostic answers after spotting gaps; keep the M/R/H agreement.
26–31	T4 - Phase 4 (26–31): Reactivation, the three congruence criteria	Congruence criteria are prior knowledge from Year 9: reactivation, not rebuilding. Table of the three criteria on the board (course notes Part 1, p. 2).	“State the three criteria for congruent triangles.”	REMAINS: selected response, anonymous code C and written connector.
31–47	T5 - Phase 5 (31–47): Conjecturing activity (hatsumon, kikan-shidō, neriage)	Project two $65^\circ/45^\circ/70^\circ$ triangles of different sizes. Have pupils measure the six sides, match those opposite equal angles and calculate the three quotients.	“What do you notice about the three quotients? Complete: if two triangles have their angles equal in pairs, are their sides proportional?”	KEEP the flawed comparison, the corrected matching and the conjecture.
47–55	T6 - Phase 6 (47–55): Partial institutionalisation + launch of drill 1.1	Re-read the kadai; have the class formulate an answer, then compare it with the definition. Launch two items from 1.1 after deferred copying.	“Are two equilateral triangles always similar?” (1.5a)	FINAL PHOTO: connect problem, responses and matome; keep the written summary.

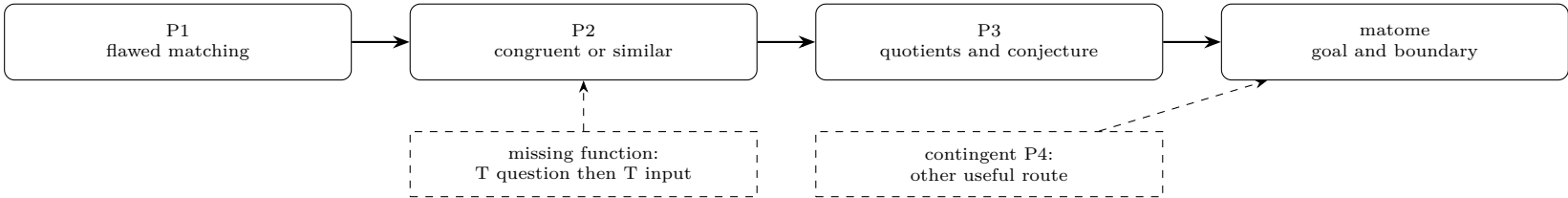
Timing consistency check: copying only starts after comparison; if time runs short, apply the following rule: Follow-up 1.5a is dropped (picked up as a flash starter in S1); the conjecturing activity and the definition are non-negotiable. The S0 variant of 01A replaces phase 5; it does not add to the timing.

3. Orchestrating the discussion: select, sequence, connect

ID	Anticipated response	Role in the narrative	Exact questions	Placement and status
P1	Sides compared in alphabetical order, including $AB/B'C'$: the ratios differ because the sides are not corresponding sides.	Highlight the need to match sides before calculating.	"Which sides did you compare? Are they opposite equal angles?"	Centre, block 1; cross out the mismatched quotients without erasing them.
P2	"The triangles are congruent": superposition fails because they have the same shape but different sizes.	Distinguish between congruent triangles and similar triangles.	"Can we superpose them? What property remains true despite the difference in size?"	Centre, block 2; cross out "congruent" and keep "same shape".
P3	All three quotients of corresponding sides are approximately 1,5; conjecture: the sides are proportional.	Move from measurements to an explicit mathematical conjecture.	"Once the sides are matched, what do the three quotients tell us together?"	Centre, block 3; keep the three quotients and the word "conjecture".
P4	"It's an enlargement with scale factor 1,5": the same multiplier transforms all three lengths.	Name the scale factor without confusing vocabulary with proof.	"What number multiplies each side of the small triangle to give its corresponding side?"	Centre, block 4; keep only if this phrasing comes up.

Planned selection order. Initial response or productive error → comparison → stabilised procedure → justification or matome. After each contribution: “Who used this idea?” then “Who can offer a more general justification?”

Essential / contingent. The function of P1-P3 is essential; the exact pupil, the wording and P4 remain contingent. On the actual board, label C1-C3 and fill in the C/P mapping afterwards.



Structured variation. V1 compare sides at random; V2 match sides opposite equal angles; V3 compare two triangles with equal angles but very different sizes. The “two fixed angles, free sizes” variant replaces phase 5; it is not added on top of it.

Invariant to verbalise. When angles are equal in pairs, corresponding sides are in the same ratio; sides are matched opposite equal angles. Measurements support a conjecture, not yet a proof.

Boundary of use. Two pairs of equal angles are enough, because the third is fixed by the 180° angle sum; a single pair of equal angles is not enough.

4. Comparing the plan with the board actually built

Observed item	Planned	Actual / evidence collected
Exploration and discussion time	0–10 test feedback; 10–15 flash quiz; 15–26 agreement and diagnostic; 26–31 congruence criteria; 31–47 exploration and neriage; 47–55 matome and launch of 1.1.	N/A
Responses that emerged	P1 flawed matching; P2 congruent/similar confusion; P3 three equal quotients and conjecture; P4 enlargement scale factor 1,5.	N/A
Order of presentation	initial / conflict → comparison → method → justification	N/A
Useful unexpected response	space P4 reserved below P3	N/A
Question that moved thinking forward	“Which sides are opposite equal angles?”; “What do the three quotients tell us together?”	N/A
Elements kept visible	problem, C1-C3, relationships, invariant, matome	N/A
Elements erased and when	auxiliary calculations only, after validation and verbalisation	N/A
Matome formulated by pupils	Equal angles: proportional corresponding sides; measuring allows us to conjecture, the proof still has to be built.	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert photograph here
or state its file name

Photograph checklist

- ☐ The problem and goal are identifiable.
- ☐ C1-C3 can be distinguished and linked to the P functions.
- ☐ A refuted error is not confused with the final result.
- ☐ Relationships and connectors are written down.
- ☐ The invariant or condition is visible.
- ☐ The matome makes sense without the teacher's spoken commentary.

Class indicator

Response or change of mind observed: _____

Deferred copying

Actual start time: _____

Local decision

Element to keep: _____

Single adjustment

Element to modify: _____

1. Before the lesson: intent and planned final board

Type: proof

Kadai “How can we prove it (not just measure it)?”**Objective**

Understand why equal angles entail proportional sides, and know that two angles are enough. This is the candidate proof for the “Proof” question on the test. Skills: reasoning, problem-solving, communicating.

Materials Course notes Part 2 (pp. 8 to 11); exercise sheet 1.3; mini-whiteboards; table of the three congruence criteria displayed.**Responses**

P1 and P2 set the boundaries; P3 constructs the copy; P4 necessarily completes the proof.

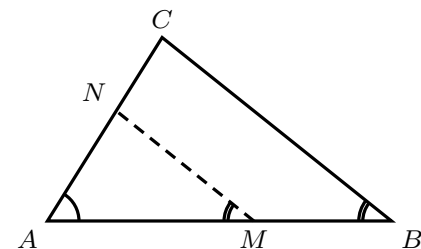
ESS essential function **CONT** contingent form **DECL** response to prompt. Actual pupil responses are labelled C1 to C4 with no name or initial; C is then linked to the P functions.

Planned final board at the end of the lesson.

IDEAL BOARD (final state): to project or reproduce

LEFT: given data and diagram

Kadai: “How can we prove it (not just measure it)?”

Given: $\widehat{A} = \widehat{A'}$, $\widehat{B} = \widehat{B'}$.Goal: $\frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC}$.If $A'B' = AB$: ASA case, so $k = 1$. Otherwise, swapping the triangles if necessary, assume $A'B' < AB$. M such that $AM = A'B'$; $(MN) \parallel (BC)$.

CENTRE: the constructed proof

X “we measure the 3 sides”: measuring is not proving1. $M \in [AB]$, $AM = A'B'$; parallel to $(BC) \rightarrow N$ 2. $\widehat{AMN} = \widehat{ABC}$ (corresponding) = $\widehat{A'B'C'}$; with $\widehat{A} = \widehat{A'}$ and $AM = A'B'$: ASA case $\Rightarrow AMN$ and $A'B'C'$ congruent so $AN = A'C'$, $MN = B'C'$ 3. nested triangles (Year 9): $\frac{AM}{AB} = \frac{AN}{AC} = \frac{MN}{BC}$
 $\Rightarrow \frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC}$ ■

RIGHT: summary (written summary)

Characterisation. Two triangles are similar if and only if their angles are equal in pairs. Two angles are enough (sum = 180°). The direction angles $\Rightarrow$ sides is proved here; the converse is proved in the course notes as an extension (exercise sheet 7.5).Model write-up (D2). “We know that $\widehat{\dots} = \widehat{\dots}$ because $\dots$. Similarly, we know that $\widehat{\dots} = \widehat{\dots}$ because $\dots$. Now, if two triangles have two pairs of equal angles, then they are similar. Therefore the triangles are similar, with corresponding vertices $\dots \leftrightarrow \dots$ ”

Physical board rule: problem, selected responses, refuted errors, connections and matome remain visible until the final photograph. Only auxiliary calculations may be erased after validation and verbalisation.

2. Storyboard: write, show, connect, keep, erase

Consistent colour code: main writing in the base colour; relationship, logical connector or invariant in an accent colour; error crossed out but still legible. Outside the auxiliary strip, no selected response is erased during the lesson.

Time	Board state	What is written	Talk and activity	Board action / status
Before entry	T0 - discreet outline	Draw three zones; reserve P1-P4 and the right-hand column.	Prepare the seating plan; do not pre-write the answer.	RESERVE spaces; everything remains open.
0–10	T1 - Phase 1 (0–10): Fluency + targeted re-activation	Complete four “third angle / similar triangles?” items: two angle calculations, two equilateral triangles, then two right-angled triangles.	“Measuring gave rise to the conjecture; what is the difference between observing and proving?”	ERASE fluency calculations; keep the conjecture and the word “proof”.
10–16	T2 - Phase 2 (10–16): “Two angles are enough” (express matome)	Write $\widehat{C} = 180^\circ - \widehat{A} - \widehat{B}$ and the same relation in the second triangle; conclude $\widehat{C} = \widehat{C}'$.	“Why are two pairs of equal angles enough, without checking the third?”	KEEP the chain of equalities and the connector “therefore”.
16–40	T3 - Phase 3 (16–40): Building THE proof (hatsumon, neriage of a proof)	Given: $\widehat{A} = \widehat{A}'$ and $\widehat{B} = \widehat{B}'$. Goal: the three ratios. Handle $A'B' = AB$, then assume $A'B' < AB$, swapping the triangles if necessary. Construct M with $AM = A'B'$, then the parallel (MN) to (BC); chain ASA and nested triangles.	“When can we place a copy of the small triangle inside the large one? Where do we place M , and how do we get N ?”	KEEP equality case, ordering assumption, construction and the three steps.
40–48	T4 - Phase 4 (40–48): Institutionalisation (matome), written summary = proof	Write the angle characterisation, the “two angles are enough” justification and a model write-up using the connectors “because, now, therefore”.	“Which sentence directly answers the kadai, and which step turns angles into proportionality?”	ALLOW copying after pupils have formulated the summary; erase nothing.
48–55	T5 - Phase 5 (48–55): 1.3 justifying similarity (the three sources of equal angles)	Work through 1.3a as a class: shared angle and corresponding angles. Launch b and c in pairs; in c, keep the crossed matching $E \leftrightarrow B$.	“For each of the two pairs of angles, which property justifies their equality?”	FINAL PHOTO: link the angle sources to the model write-up.

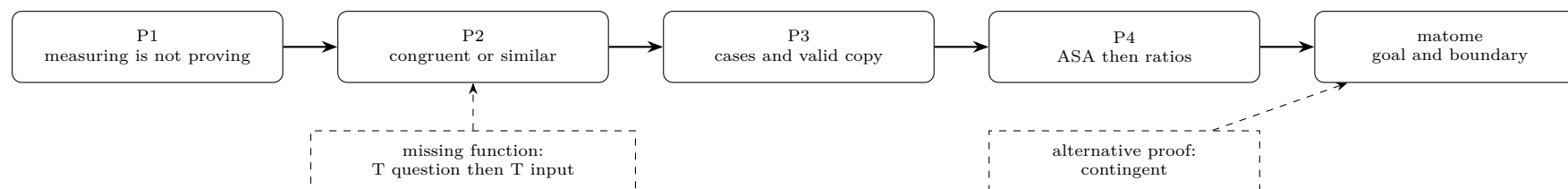
Timing consistency check: copying only starts after comparison; if time runs short, apply the following rule: 1.3 moves entirely to homework (the model write-up on the board for a) is non-negotiable); steps 1–3 of the proof are never cut.

3. Orchestrating the discussion: select, sequence, connect

ID	Anticipated response	Role in the narrative	Exact questions	Placement and status
P1	"We measure the three sides and check."	Distinguish an experimental check from a general proof.	"Can three measurements prove a property for all triangles?"	Centre, block 1; cross out "proof", keep "conjecture".
P2	"The triangles are congruent." Yet the scale factor is not necessarily 1.	Rule out the confusion between congruence and similarity before constructing.	"If $k \neq 1$, can we superpose the two triangles?"	Centre, block 2; refute and keep as a safeguard.
P3	After the case $A'B' = AB$, assume $A'B' < AB$ by swapping the triangles if necessary, then choose $M \in [AB]$ such that $AM = A'B'$.	Make the construction valid in all cases, with no hidden assumption.	"Why can we place M on $[AB]$? What do we do if $A'B' > AB$?"	Centre, block 3; keep the equality case, the possible swap and the construction.
P4	Draw the line through M parallel to (BC) ; ASA gives AMN and $A'B'C'$ congruent, then the nested triangles give the ratios.	Combine a congruence criterion and proportionality into a complete proof.	"Why are AMN and $A'B'C'$ congruent, and which property then gives the three ratios?"	Centre, block 4; connect with "therefore" to the final ratios.

Planned selection order. Initial response or productive error $\rightarrow$ comparison $\rightarrow$ stabilised procedure $\rightarrow$ justification or matome. After each contribution: "Who used this idea?" then "Who can offer a more general justification?"

Essential / contingent. Functions P1-P4 are essential to the narrative: P3 makes the construction legitimate and P4 completes the proof. The exact pupils, the precise wording and any alternative route remain contingent. On the actual board, label the C responses and fill in their mapping to P afterwards.



Structured variation. V1 two pairs of equal corresponding angles; V2 copy of the small triangle placed inside the large one; V3 parallel line giving an ASA case then a nested configuration. We move from angle information to proportionality.

Invariant to verbalise. Two pairs of equal angles fix the third. After the case $k = 1$, we place the smaller triangle inside the larger one; ASA gives a triangle congruent to the small one, and the proportionality of nested triangles then gives the three ratios of corresponding sides.

Boundary of use. A measurement, however accurate, does not prove the theorem; the angle characterisation requires two pairs of equal angles, not just one. The converse is proved in the course notes as an extension (exercise sheet 7.5).

4. Comparing the plan with the board actually built

Observed item	Planned	Actual / evidence collected
Exploration and discussion time	0–10 fluency; 10–16 third angle; 16–40 construction and neriage of the proof; 40–48 matome; 48–55 application 1.3.	N/A
Responses that emerged	P1 measurement mistaken for proof; P2 congruent/similar confusion; P3 copy of the small triangle; P4 parallel line, ASA and nested ratios.	N/A
Order of presentation	initial / conflict → comparison → method → justification	N/A
Useful unexpected response	space reserved below P4 for an alternative proof, if one emerges	N/A
Question that moved thinking forward	“Do three measurements prove the property for all triangles?”; “Which construction links ASA to the side ratios?”	N/A
Elements kept visible	problem, C1-C4, relationships, invariant, matome	N/A
Elements erased and when	auxiliary calculations only, after validation and verbalisation	N/A
Matome formulated by pupils	Two pairs of equal angles are enough; once copying the small triangle inside the large one is justified, ASA and nested triangles give all three ratios. The converse is proved in the course notes as an extension (exercise sheet 7.5).	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert photograph here
or state its file name

Photograph checklist

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Class indicator

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Deferred copying

Actual start time: _____

Local decision

Element to keep: _____

Single adjustment

Element to modify: _____

1. Before the lesson: purpose and planned final board

Type: guided practice and consolidation of a method

Kadai	“Calculating an inaccessible length using similarity”	Objective	Calculate lengths with the match / set up / solve method; recognise the bow-tie configuration. Skills: calculating, modelling, communicating.
Equipment	Exercise sheet 1.1, 1.2, 1.4, 2.2; corresponding teacher instructions; timer.	Pupil work	P1-P4 are four comparable stages of the written solution to 2.2; P4 is the complete reference solution.
ESS essential function	CONT contingent form	DECL work to elicit. Actual pupil work is labelled C1 to C4 with no name or initial; C is then linked to the functions P.	

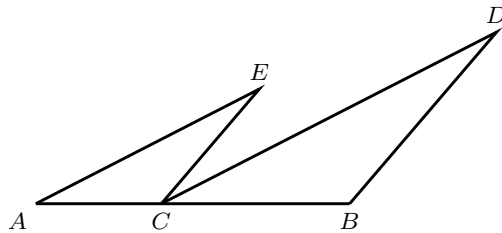
Planned final board at the end of the slot.

IDEAL BOARD (final state): to project or reproduce

LEFT: problem 2.2 + drill corner

Kadai: “Calculating an inaccessible length using similarity”

$C \in [AB]$; $AC = 3$; $AB = 7,5$; $BD = 5,4$; $CD = 9$; $(AE) \parallel (CD)$; $(CE) \parallel (BD)$.



Drill corner: 1.1: a) $47/86 \rightarrow 47$: similar ✓; e) isosceles 92° : base 44° .

1.4 bow-tie: vertically opposite + alternate interior; $k = \frac{4,8}{3,6} = \frac{4}{3}$; $AE = 3,2$; $BC = 3,9$; ✗ $BC \approx 6,9$ (impossible: $BC < DE$), crossed out.

CENTRE: 2.2, the climax journey

a) $(AE) \parallel (CD)$ cut by $(AB) \Rightarrow \widehat{CAE} = \widehat{BCD}$ (corresponding)

b) $(CE) \parallel (BD) \Rightarrow \widehat{ACE} = \widehat{CBD}$; two pairs of equal angles $\Rightarrow$ similar ($A \leftrightarrow C$, $C \leftrightarrow B$, $E \leftrightarrow D$)

✗ $AB = 7,5$ placed in the table (CB forgotten): crossed out

✗ matching “in letter order”: crossed out

c) ✓ $CB = 7,5 - 3 = 4,5$ (first!)

CBD	$CB = 4,5$	$CD = 9$	$BD = 5,4$
ACE	$AC = 3$	$AE = ?$	$CE = ?$

$k = \frac{3}{4,5} = \frac{2}{3}$; $AE = 6$; $CE = 3,6$

check: $\frac{3}{4,5} = \frac{6}{9} = \frac{3,6}{5,4} = \frac{2}{3}$, full written solution copied (the reference)

RIGHT: summary (the written summary)

Match: using the angles (never the letters). Opposite the largest angle lies the longest side.

Set up: table of corresponding sides, starting triangle in the top row, target triangle in the bottom row.

Solve: find k from a complete column, then $\times k$ towards the target row; check the ratios (model L1).

Practical rule: the problem, selected pupil work, refuted errors, links and matome remain visible until the final photograph. Only auxiliary calculations may be erased after validation and verbalisation.

2. Storyboard: write, show, link, keep, erase

Consistent colour code: main writing in the base colour; relationship, logical connector or invariant in an accent colour; error crossed out but still legible. Outside the auxiliary strip, no selected pupil work is erased during the lesson.

Time	Board state	What is written	Speech and activity	Board action / next step
Before entry	T0 - discreet outline	Draw three zones; reserve space for P1-P4 and the right-hand zone.	Prepare the classroom layout; do not write the answer in advance.	RESERVE the spaces; keep everything available.
0–10	T1 - Phase 1 (0–10): Fluency 1.1 (6 min) + targeted review of 1.2b	Cover six angle-based identifications, then revisit the exact counter-example from the homework: $\frac{1}{5} = \frac{1,6}{8} = 0,2$ but $\frac{1,2}{6,5} \neq 0,2$.	“Are two equal ratios enough to conclude that the triangles are similar?”	ERASE the fluency work; keep the three ratios from 1.2b and the conclusion “no”.
10–16	T2 - Phase 2 (10–16): Using a real error from the homework	Anonymise a piece of pupil work that concludes without calculating the third angle. Take a vote, locate the gap, then have pupils rephrase the complete criterion.	“What is missing from this reasoning in order to conclude?”	KEEP the faulty line, the missing piece of data and the correction.
16–30	T3 - Phase 3 (16–30): 1.4 the bow-tie, using angles	The intercept theorem (Thales’ theorem in France) is forbidden (chapter 01C): here we prove using angles. Diagram on the board, name the two triangles (<i>ABC</i> at the top, <i>ADE</i> at the bottom).	“Two pairs of equal angles in the bow-tie: which ones, and why?”	REMAINS: selected pupil work, anonymous code C and written connector.
30–52	T4 - Phase 4 (30–52): 2.2 the brevet (French national exam at the end of Year 10) exercise, in depth (the climax, the reference solution)	Compare P1 to P4: only one pair of angles; two pairs but wrong matching; correct calculation without a check; complete solution with $CB = 4,5$, corresponding sides, $k = \frac{2}{3}$, $AE = 6$, $CE = 3,6$ and three ratios.	“Which line is missing between each solution and the next to make the reasoning verifiable?”	KEEP P1-P4 and link each fix to the final method.
52–55	T5 - Phase 5 (52–55): Cliffhanger	Display the diagram from 2.5, with no parallel lines and only one known pair of angles. Leave the question open for S3.	“With no parallel lines and only one angle, what other criterion could prove similarity?”	FINAL PHOTO: keep the diagram and the question, without giving the answer.

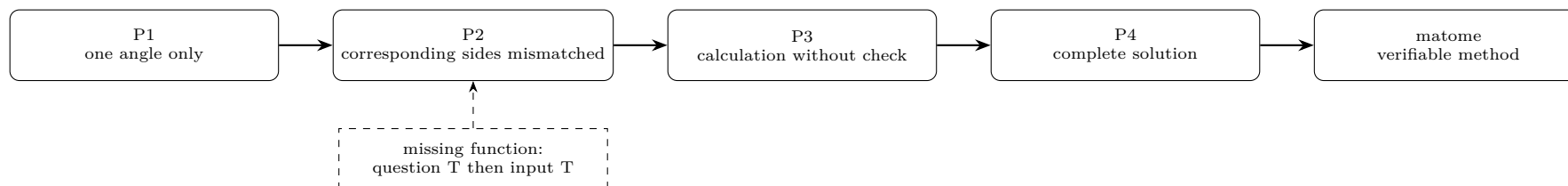
Timing consistency check: copying only begins after comparison; if time runs short, apply the following rule: The final write-up of 2.2 moves to the opening of S3; 1.4 (the bow-tie using angles) and questions a–b of 2.2 cannot be cut.

3. Leading the discussion: select, order, connect

ID	Anticipated pupil work	Role in the lesson flow	Exact questions	Placement and next step
P1	After $\widehat{CAE} = \widehat{BCD}$, the solution already concludes that ACE and CBD are similar.	Show that a single pair of angles is not enough.	"Which second pair of angles must be established, and using which parallel lines?"	Centre, block 1; cross out the premature conclusion without erasing it.
P2	Both pairs of angles are justified, but the solution matches the sides "in letter order", for instance $[AC]$ with $[CD]$.	Separate the proof of similarity from the matching of corresponding sides.	"Which vertices correspond according to the angles, rather than the letter order?"	Centre, block 2; keep the corrected correspondence $A \leftrightarrow C$, $C \leftrightarrow B$, $E \leftrightarrow D$.
P3	$CB = 7,5 - 3 = 4,5$, correct table, $k = \frac{2}{3}$, $AE = 6$ and $CE = 3,6$, but no final check.	Consolidate match / set up / solve, then highlight what is still missing from a verifiable solution.	"Do the results obey the same ratio across all three columns?"	Centre, block 3; keep the calculations and leave space for the check line.
P4	Complete solution: two justified equalities of angles, correspondence, $CB = 4,5$, table, $k = \frac{2}{3}$, $AE = 6$, $CE = 3,6$ and $\frac{3}{4,5} = \frac{6}{9} = \frac{3,6}{5,4}$.	Become the reusable reference solution for the twin brevet exercises.	"Which line in P4 allows the marker to check each of the steps?"	Centre, block 4; keep in full and link to the matome.

Planned selection order. Initial answer or productive error $\rightarrow$ comparison $\rightarrow$ consolidated procedure $\rightarrow$ justification or matome. After each contribution: "Who used this idea?" then "Who can suggest a more general justification?"

Essential / contingent. Functions P1-P4 are essential: P4 is the verifiable outcome of P1-P3. The exact authors and wording are contingent. If a piece of work is missing, ask a public question T; if the function is still absent, write up a teacher contribution T linked to 2.2.



Structured variation. The bow-tie serves as a guided stepping stone; in 2.2, V1 a single pair of angles, V2 two pairs but mismatched corresponding sides, V3 correct calculation without a check, V4 complete solution.

Invariant to elicit verbally. Justify similarity, match using the angles, calculate any intermediate length, set up one triangle per row, use a single scale factor k , then check all three quotients.

Scope of validity. Two equal ratios are not enough to prove similarity; calculating lengths is only permitted once similarity has been justified and corresponding sides have been matched.

4. Comparing the plan with the board actually built

Observed item	Planned	Actual / evidence gathered
Exploration and whole-class discussion time	0–10 fluency; 10–16 real error; 16–30 bow-tie; 30–52 reference solution 2.2; 52–55 open question for S3.	N/A
Pupil work that emerged	P1 conclusion after a single angle; P2 mismatched corresponding sides; P3 correct calculation without check; P4 complete and checked solution to 2.2.	N/A
Order of presentation	initial / conflict → comparison → method → justification	N/A
Useful unexpected pupil work	space reserved below P4 for another correct check	N/A
Question that moved the lesson forward	(on 1.2b) “Are two equal ratios enough?”; (on 2.2) “Which line is missing between this solution and the next?”	N/A
Elements kept visible	problem, C1-C4, links, invariant, matome	N/A
Elements erased and when	auxiliary calculations only, after validation and verbalisation	N/A
Matome phrased by the pupils	Justify similarity, match using the angles, calculate intermediate lengths, set up corresponding sides, solve with a single scale factor and check the three quotients.	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert the photograph here
or state its filename

Photograph checklist

- ☐ The problem and the goal can be identified.
- ☐ C1-C4 can be distinguished and linked to functions P.
- ☐ A refuted error cannot be confused with the final result.
- ☐ Relationships and connectors are written down.
- ☐ The invariant or condition is visible.
- ☐ The matome makes sense without the teacher’s spoken commentary.

Class indicator	Observed work or change of mind: _____	Deferred copying	Actual start time: _____
Local decision	Element to keep: _____	Single adjustment	Element to modify: _____

Bansho keikaku - planned board

1. Before the lesson: purpose and planned final board

Type: practice and variation

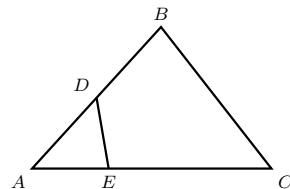
Kadai	“Similar with only one known angle?”	Objective	Match sides in tricky configurations (anti-parallel); use the third criterion when only one angle is known; prove by contradiction that two lines are not parallel (2.5 c); settle a true/false question with a counterexample. Skills: calculating, reasoning, communicating.
Equipment	Exercise sheet 2.3, 2.5, 1.5; teacher instructions; mini-whiteboards.	Pupil work	P1–P4 planned and essential: choice of method, matching, SAS criterion, then proof by contradiction.
ESS	essential function	CONT	contingent form
DECL	work to elicit. Actual pupil work is labelled C1, C2, C3, C4 with no name or initial; C is then linked to the functions P.		

Planned final board at the end of the slot.

IDEAL BOARD (final state): to project or reproduce

LEFT: drill corner + the 2.3 trap + the vote

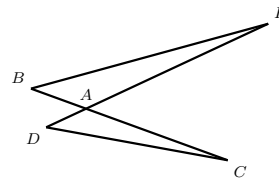
Kadai: “Similar with only one known angle?”

Drill corner: 2.1: two methods side by side ($k = 1,5$ / cross-multiplication).2.3 config 2: $\widehat{ADE} = \widehat{ACB}$; $AD = 4$, $AC = 10$, $AB = 8$, $CB = 7,5$.✗ $[AD] \leftrightarrow [AB]$ (the letters): crossed out✓ $[AD] \leftrightarrow [AC]$, $[DE] \leftrightarrow [CB]$; $k = \frac{2}{5}$; $DE = 3$, $AE = 3,2$

1.5 vote: a T · b F · c F · d T · e F · f F · g T

e) 80° : at the apex (80/50/50) or at the base (80/80/20)

CENTRE: 2.5, the bow-tie without parallel lines (the climax)

 (BC) and (DE) intersect at A ; $AB = 28$, $AC = 72$, $AD = 21$, $AE = 96$; no parallel lines given.

✗ “nothing can be done: no parallel lines” (the alternate-interior reflex hits a dead end)

✗ matching AB with AE (two sides of the same triangle): crossed outa) ✓ vertically opposite at A ; $\frac{AB}{AD} = \frac{28}{21} = \frac{4}{3} = \frac{96}{72} = \frac{AE}{AC}$: SAS for similarity (D3) $\Rightarrow DAC$ and BAE similar ($D \leftrightarrow B$, $A \leftrightarrow A$, $C \leftrightarrow E$)c) by contradiction: if $(DC) \parallel (BE)$, then $\widehat{DCA} = \widehat{EBA}$ (alternate interior); yet (a): $\widehat{DCA} = \widehat{BEA}$; ABE would be isosceles at A , impossible: $AB = 28 \neq 96 = AE$. Not parallel. ■b) left open: lengths $\times \frac{4}{3}$, and the area, multiplied by what? $\rightarrow$ next lesson (k^2).

RIGHT: summary (the written summary)

Match using the equal angles, never the position of the letters.

3rd criterion (SAS for similarity, D3): an equal angle included between two pairs of proportional sides $\Rightarrow$ similar.

Reasoning by contradiction: assume the statement, follow the steps, reach a contradiction, so the assumption was false.

In 2.5 c): $(DC) \parallel (BE) \Rightarrow \widehat{DCA} = \widehat{EBA}$; similarity $\Rightarrow \widehat{DCA} = \widehat{BEA}$; hence $\widehat{EBA} = \widehat{BEA}$: ABE isosceles at A .

The largest angle is opposite the longest side.

Practical rule: the problem, selected pupil work, refuted errors, links and matome remain visible until the final photograph. Only auxiliary calculations may be erased after validation and verbalisation.

2. Storyboard: write, show, link, keep, erase

Consistent colour code: main writing in the base colour; relationship, logical connector or invariant in an accent colour; error crossed out but still legible. Outside the auxiliary strip, no selected pupil work is erased during the lesson.

Time	Board state	What is written	Speech and activity	Board action / next step
Before entry	T0 - discreet outline	Draw three zones; reserve space for P1–P4 and the right-hand zone.	Prepare the classroom layout; do not write the answer in advance.	RESERVE the spaces; keep everything available.
0–8	T1 - Phase 1 (0–8): Fluency + correction of 2.1	Run two quick matching drills, then place the calculation using $k = 1,5$ and cross-multiplication side by side.	“When is the scale factor quicker, and why does cross-multiplication remain general?”	ERASE the calculations; keep the criterion for choosing between the two methods.
8–22	T2 - Phase 2 (8–22): 2.3, both configurations (including the anti-parallel trap)	Config. 1: keep only the three ratios. Config. 2: take a vote on the side corresponding to DE , then write $AD \leftrightarrow AC$, $AE \leftrightarrow AB$, $DE \leftrightarrow CB$ and $k = \frac{2}{5}$.	“In the anti-parallel configuration, which angles dictate the matching for DE ?”	CROSS OUT letter-order matching; keep corresponding sides and the check $k < 1$.
22–40	T3 - Phase 3 (22–40): 2.5 in depth, SAS then contradiction (the climax)	Display the bow-tie without parallel lines. Compare the two ratios around the vertically opposite angle; conclude using SAS. For c, assume the lines are parallel and follow the steps to $AB = AE$, contradicting $28 \neq 96$.	“Which criterion uses an equal angle included between two pairs of proportional sides?”	KEEP the assumption, the consequences and the contradiction of the proof by contradiction.
40–52	T4 - Phase 4 (40–52): 1.5 true/false vote (level R of the test)	Take a vote before justifying. Require a counterexample for every false statement; compare the two possible positions of the 80° angle in an isosceles triangle.	“And if the angle were 100° instead of 80° : could it still be at the base?”	KEEP both arrangements for 80° and the impossibility for 100° .
52–55	T5 - Phase 5 (52–55): Protected matome	Re-read the kadai; gather two phrasings; write the shared answer, then compare it with the official written summary: one equal angle and two pairs of proportional sides.	“With only one known angle, what information makes similarity provable?”	PROTECT these three minutes; link the kadai to SAS and keep the matome for the final photograph.

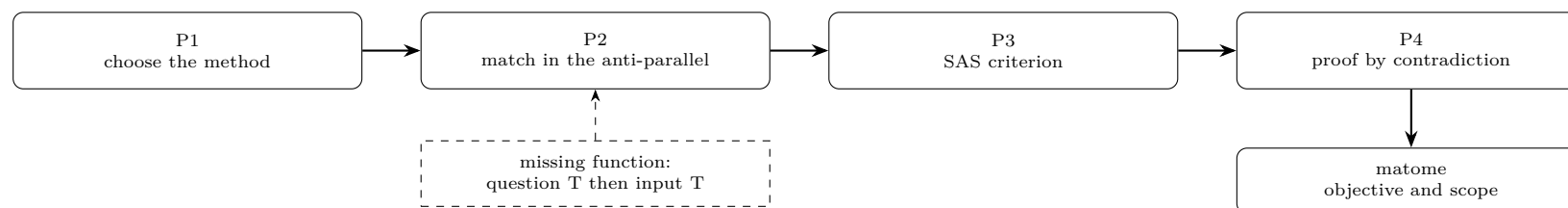
Timing consistency check: minutes 52–55 belong to the matome and must not spill over into another phase. If time runs short, restrict the vote on 1.5 to items e) and g). Configuration 2 of 2.3, the SAS criterion in 2.5 a), the proof by contradiction in 2.5 c) and the matome cannot be cut.

3. Leading the discussion: select, order, connect

ID	Anticipated pupil work	Role in the lesson flow	Exact questions	Placement and next step
P1	The scale factor is convenient when k is simple, such as $\frac{3}{2}$ or $\frac{2}{3}$; cross-multiplication works for any value of a defined ratio.	Compare the two methods; on your paper, calculate using the scale factor k (model L1), and use cross-multiplication only if the problem asks for it.	"Which method takes fewer steps here, and which one remains available for any defined ratio?"	Centre, block 1; keep both approaches and when each is most convenient.
P2	Corresponding sides $AD \leftrightarrow AC$, $AE \leftrightarrow AB$, $DE \leftrightarrow CB$; $k = \frac{2}{5}$, so $DE = 3$ and $AE = 3, 2$. The matching $AD \leftrightarrow AB$ is wrong.	Give matching by angles priority over letter order.	"Which angles justify each pair of corresponding sides?"	Centre, block 2; cross out the wrong matching and keep the check $k < 1$.
P3	$\widehat{DAC} = \widehat{BAE}$ and $\frac{AB}{AD} = \frac{AE}{AC} = \frac{4}{3}$; SAS for similarity applies. Comparing AB with AE would mean comparing two sides of the same triangle.	Bring out the SAS criterion when parallel lines are absent.	"Around the equal angle, which side of one triangle should be compared with which side of the other?"	Centre, block 3; keep the angle, the two ratios and the conclusion.
P4	If $(DC) \parallel (BE)$, the angles would lead to $\widehat{EBA} = \widehat{BEA}$, hence $AB = AE$; yet $28 \neq 96$. The lines are therefore not parallel.	Give a visible structure to the proof by contradiction.	"Suppose the lines are parallel: which consequence contradicts the given lengths?"	Centre, block 4; keep the assumption, the contradiction and the conclusion.

Planned selection order. Initial answer or productive error $\rightarrow$ comparison $\rightarrow$ consolidated procedure $\rightarrow$ justification or matome. After each contribution: "Who used this idea?" then "Who can suggest a more general justification?"

Essential / contingent. The functions of P1–P4 are essential; only the exact author and wording used are contingent. If the class does not produce the next step, ask a public question T and then mark any teacher input with T. On the actual board, write C1–C4 and then fill in the C/P correspondence.



Structured variation. V1 separate triangles; V2 anti-parallel configuration; V3 bow-tie without parallel lines, handled using an included angle and two ratios. The diagram changes, but matching by angles remains.

Invariant to elicit verbally. The SAS criterion requires an equal angle included between two pairs of proportional sides. A proof by contradiction assumes the statement, deduces a contradiction, and then rejects the assumption.

Scope of validity. A single equal angle is not enough without proportionality of the two sides enclosing it. For an isosceles triangle, 80° can occupy two positions, whereas 100° cannot be a base angle.

4. Comparing the plan with the board actually built

Observed item	Planned	Actual / evidence gathered
Exploration and whole-class discussion time	0–8 choice of method; 8–22 configurations of 2.3; 22–40 SAS and contradiction; 40–52 true/false vote; 52–55 protected matome.	N/A
Pupil work that emerged	P1 scale factor vs cross-multiplication comparison; P2 anti-parallel matching; P3 SAS criterion; P4 proof by contradiction of non-parallelism.	N/A
Order of presentation	initial / conflict → comparison → method → justification	N/A
Useful unexpected pupil work	another valid contradiction, if one emerges	N/A
Question that moved the lesson forward	“Around the equal angle, which sides should be compared?”; “If the lines were parallel, what contradiction would we get?”	N/A
Elements kept visible	problem, C1–C4, links, invariant, matome	N/A
Elements erased and when	auxiliary calculations only, after validation and verbalisation	N/A
Matome phrased by the pupils	In every configuration, we match using the angles. With a single angle, SAS requires two equal ratios; proof by contradiction proves a negation via a contradiction.	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert the photograph here
or state its filename

Photograph checklist

- ☐ The problem and the goal can be identified.
- ☐ C1–C4 can be distinguished and linked to functions P.
- ☐ A refuted error cannot be confused with the final result.
- ☐ Relationships and connectors are written down.
- ☐ The invariant or condition is visible.
- ☐ The matome makes sense without the teacher's spoken commentary.

Class indicator	Observed work or change of mind: _____	Deferred copying	Actual start time: _____
Local decision	Element to keep: _____	Single adjustment	Element to modify: _____

Bansho keikaku - planned board

1. Before the lesson: aim and planned final board

Type: conjecture, proof and institutionalisation

Kadai	“Lengths, areas, volumes: multiplied by what?”	Objective	Conjecture the general rule for areas, prove it on a rectangle; know the k , k^2 , k^3 effects and use them both ways. Skills: investigating, reasoning, calculating.
Materials	Course notes part 4 (pp. 19–22); exercise sheet 3.2 (activity), 3.1 (drill), 3.3; mini-whiteboards; ruler.	Pupil work	P1–P4 planned and essential: $\times k$ error, measurement, proof on the rectangle, then direct and reverse transfer.
ESS	essential function	CONT	contingent form
DECL	work to elicit. Actual pupil work is labelled C1, C2, C3, C4 without names or initials; C is then linked to the P functions.		

Planned final board at the end of the slot.

IDEAL BOARD (final state): to project or reproduce

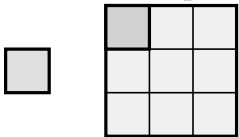
LEFT: the conjecture (3.2)

Kadai: “Lengths, areas, volumes: multiplied by what?”

Quadrilateral on a grid, diagonal $[BD]$; area $ABD = 12$.

Reduction $\times 0,5$: area =?

Seen on a square $\times 3$:



side 1, area 1 $\rightarrow$ side 3, area 9 = 3^2

CENTRE: the proof

✗ “area $\times 0,5$ ”: false ($3 \neq 6$)

✓ $\mathcal{A}' = 3 = 12 \times 0,25 = 12 \times 0,5^2$

Proof (rectangle):

$\mathcal{A}' = (kL)(k\ell) = k \times k \times L\ell$
 $= k^2 \mathcal{A}$ (two dimensions)

1 dimension $\rightarrow k$; 2 $\rightarrow k^2$; 3 $\rightarrow k^3$

S3 cliffhanger resolved: 2.5 b): lengths $\times \frac{4}{3} \Rightarrow$
 areas $\times \left(\frac{4}{3}\right)^2 = \frac{16}{9}$ ✓

RIGHT: summary (written record)

Scale factor k (A2): lengths $\times k$; angles preserved; areas $\times k^2$; volumes $\times k^3$.

Red line: the area factor is k^2 (equal to k only if $k = 1$).

Reverse direction (A3): area ratio = k^2 , then find $k > 0$.

Physical board rule: problem, selected pupil work, refuted errors, connections and matome remain visible until the final photograph. Only auxiliary calculations may be erased after validation and verbalisation.

2. Storyboard: write, show, connect, keep, erase

Consistent colour code: main writing in the base colour; relationship, logical connector or invariant in an accent colour; error crossed out but legible. Outside the auxiliary strip, no selected pupil work is erased during the lesson.

Time	Board state	What is written	Speech and activity	Board action / status
Before entry	T0 - discreet outline	Draw three zones; reserve P1–P4 and the right-hand side.	Prepare the seating plan; do not pre-write the answer.	RESERVE spaces; everything remains available.
0–10	T1 - Phase 1 (0–10): Starter + differentiated weekend feedback	Do starter 1.3b. Depending on the assigned work, compare two scripts of 6.1 or two error log entries on matching; apply the same marking grid.	“Would these two scripts get full marks? Which written step separates them?”	KEEP only the writing gap and the grid rule.
10–24	T2 - Phase 2 (10–24): Activity 3.2, conjecturing k and k^2	On the grid, measure the initial area 12 and that of the reduction with scale factor 0,5. Keep the error “area multiplied by 0,5” and the observed value 3.	“Which exponent completes $\mathcal{A}' = 0,5^{\square} \mathcal{A}$, since $3 = 12 \times 0,25$?”	KEEP error, measurement and conjecture $\mathcal{A}' = k^2 \mathcal{A}$.
24–34	T3 - Phase 3 (24–34): Proof of k^2 (matome) + institutionalisation	Proof on the board: rectangle $L \times \ell$ enlarged with scale factor $k \rightarrow (kL) \times (k\ell) = k^2 \mathcal{A}$; then the square of side 1 enlarged $\times 3$ contains $9 = 3^2$ grid squares.	“Why $\times k^2$ for the area, and not $\times k$?”	REMAINS until the end; allow copying after comparison.
34–44	T4 - Phase 4 (34–44): Timed drill 3.1 (8 min)	Mark the eight items; distinguish k , k^2 and k^3 . Keep item h: angles are preserved, not multiplied.	“For each quantity, which exponent of k matches its number of dimensions?”	ERASE calculations; keep the length/area/volume/angles table.
44–55	T5 - Phase 5 (44–55): 3.3 reverse direction (test level)	“ k^2 first, k next.”	“Question a: justify similarity before calculating the area.”	FINAL PHOTO: connect problem, pupil work and matome; keep the record.

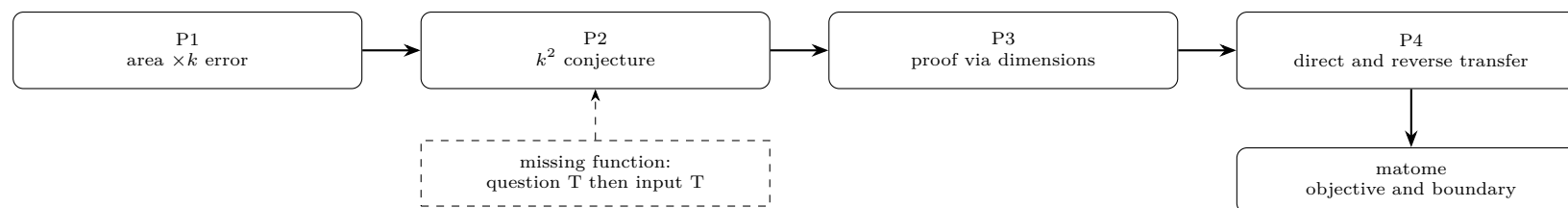
Time consistency check: copying only starts after comparison; if time runs short, apply the following rule: Drill 3.1 moves to the opening fluency of S5; 3.3 a) (similarity before area) cannot be cut; the proof of k^2 is never cut (candidate for Ex. 4B on the test).

3. Managing the discussion: select, sequence, connect

ID	Anticipated pupil work	Role in the narrative	Exact questions	Placement and status
P1	"The lengths are divided by 2, so the area is too": $12 \times 0,5 = 6$.	Expose the confusion between a length and an area, before any institutionalisation.	"If this rule were true, what area would it predict? What does the grid show?"	Centre, block 1; cross out 6 only after confrontation, leaving it legible.
P2	$A' = 3 = 12 \times 0,25 = 12 \times 0,5^2$; the $\times 0,5$ hypothesis is refuted by the grid.	Bring out the exponent 2 from a measurable confrontation.	"0,25 is what power of 0,5?"	Centre, block 2; keep error, measurement and conjecture.
P3	$(kL)(k\ell) = k^2 L\ell$: both dimensions of the rectangle are multiplied by k .	Prove the rule algebraically on a rectangle, then institutionalise its extension to areas.	"How many factors of k appear in the area of the rectangle?"	Centre, block 3; keep the algebraic proof and its scope.
P4	After justifying similarity, $k = \frac{4}{3}$ and the area is $2,7 \left(\frac{4}{3}\right)^2 = 4,8 \text{ cm}^2$. In the reverse direction, $k^2 = 25$ gives $k = 5 > 0$.	Transfer the rule to direct calculation and the reverse direction without forgetting the similarity condition.	"What is the ratio of lengths before touching the areas, and why do we take the positive square root on the way back?"	Centre, block 4; keep the justification and the meaning of k .

Planned selection order. Initial answer or productive error $\rightarrow$ comparison $\rightarrow$ stabilised procedure $\rightarrow$ justification or matome. After each contribution: "Who used this idea?" then "Who can suggest a more general justification?"

Essential / contingent. The functions of P1–P4 are essential; only the exact author and the wording used are contingent. If a step is missing, ask a public question T and then sign any teacher input with T. On the actual board, write C1–C4 and later fill in the C/P correspondence.



Structured variation. V1 reduction with scale factor 0,5 on a grid; V2 enlargement with scale factor 3 of a square; V3 any scale factor k on a rectangle. The number changes, the number of dimensions fixes the exponent.

Invariant to elicit. For a similarity scale factor $k > 0$, lengths are multiplied by k , angles are preserved, areas by k^2 and volumes by k^3 .

Scope of validity. These laws assume that the figures or solids are similar. In the reverse direction, k is positive: if the ratio of areas is r , then $k = \sqrt{r}$.

4. Comparing the plan with the board actually built

Observed item	Planned	Actual / evidence collected
Exploration and whole-class discussion time	0–10 starter and comparison; 10–24 conjecture on the grid; 24–34 proof of k^2 ; 34–44 drill; 44–55 reverse direction.	N/A
Pupil work that emerged	P1 area $\times k$; P2 area 3 and k^2 conjecture; P3 proof on the rectangle; P4 direct application and reverse direction.	N/A
Order of presentation	initial / conflict $\rightarrow$ comparison $\rightarrow$ method $\rightarrow$ justification	N/A
Useful unexpected work	another valid justification by cutting or tiling, if it appears	N/A
Question that moved thinking forward	“0,25 is what power of 0,5?”; “How many factors of k appear in an area?”	N/A
Elements left visible	problem, C1–C4, connections, invariant, matome	N/A
Elements erased and when	auxiliary calculations only, after validation and verbalisation	N/A
Matome stated by the pupils	One dimension gives k , two give k^2 , three give k^3 ; angles remain unchanged and similarity must be justified first.	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert the photograph here
or state its file name

Photograph check

- ☐ The problem and the goal are identifiable.
- ☐ C1–C4 are distinguishable and linked to the P functions.
- ☐ A refuted error is not confused with the result.
- ☐ Relationships and connectors are written.
- ☐ The invariant or condition is visible.
- ☐ The matome makes sense without the teacher’s spoken commentary.

Class indicator	Observed work or change of mind: _____	Delayed copying	Actual start time: _____
Local decision	Element to keep: _____	Single adjustment	Element to modify: _____

Bansho keikaku - planned board

1. Before the lesson: aim and planned final board

Type: problem posing, modelling and critical thinking

Kadai	“Creating a problem from a solved problem”	Objective	Create a problem under constraints, have peers solve it and revise it (the peak of the lesson); refute a claim by calculation (critical thinking); tackle a scale problem using the Wheel. Skills: investigating, modelling, reasoning, communicating.
Materials	Exercise sheet 3.4, 3.5, 5.2, 5.3, 4.1; teacher instructions; Wheel poster; worked solutions for the exercise sheet (to hand out).	Pupil work	P1–P4 planned and essential: ambiguity, inconsistency, solvable problem, then explicit check.
ESS essential function	CONT contingent form	DECL work to elicit. Actual pupil work is labelled C1, C2, C3, C4 without names or initials; C is then linked to the P functions.	

Planned final board at the end of the slot.

IDEAL BOARD (final state): to project or reproduce

LEFT: drill corner + critical thinking + park

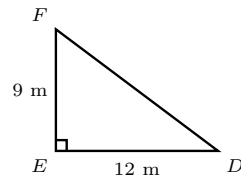
Kadai: “Creating a problem from a solved problem”

Drill corner: 5.2 without calculating: $3^2 = 9$ (not 6); a reduction shortens; largest angle $\leftrightarrow$ longest side; area $\div 100^2$.
 3.4: $k = \frac{1}{3}$, area 1 cm^2 (check: $k^2 = \frac{1}{9}$).

3.5 statue, model: $k = \frac{1}{4}$; same materials and internal geometry assumed:
 $225/64 \approx 3,5 \text{ t} \neq 14 \text{ t}$: full model refuted, outer shape undecidable.

~~X~~ $225 \div 4 \approx 56 \text{ t}$ (k instead of k^3): crossed out

4.1 park (sketch, Wheel):



scale $\frac{6}{1200} = \frac{1}{200}$; actual area 54 m^2 ;
 e) area $\div 200^2 = 40\,000$: $13,5 \text{ cm}^2$ (two routes)

CENTRE: the “modify, create, have peers solve” cycle (the peak)

5.3, the model: vertically opposite at O + $\widehat{OAB} = \widehat{OCD} \Rightarrow$ similar; $k = \frac{4}{6} = \frac{2}{3}$; $OD = 3,6$; $AB = 7,5$

Modify ($OC : 4 \rightarrow 3$), predicted before calculating: k decreases $\Rightarrow OD$ decreases, AB increases; checked: $k = \frac{1}{2}$; $OD = 2,7$; $AB = 10$

Create (the second triangle is a reduction of the first with scale factor $\frac{3}{4}$; one unknown per triangle), two pair-written problems displayed, compared:

~~X~~ vertex correspondence missing (left to guesswork): to revise

~~X~~ one given value contradicts the ratio $\frac{3}{4}$: to revise

✓ solved by a peer, checked (three ratios, large/small consistency) $\rightarrow$ the revised problem is adopted

RIGHT: summary (written record)

A calculation can refute a claim: that is critical thinking (V3).

The Wheel: what I KNOW / what I WANT / my TOOLS / my CHECK, no calculation before TOOLS.

A good problem: solvable · sufficient data · consistent · verifiable.

Reduction % do not carry across dimensions (area $\times k^2$, volume $\times k^3$).

Physical board rule: problem, selected pupil work, refuted errors, connections and matome remain visible until the final photograph. Only auxiliary calculations may be erased after validation and verbalisation.

2. Storyboard: write, show, connect, keep, erase

Consistent colour code: main writing in the base colour; relationship, logical connector or invariant in an accent colour; error crossed out but legible. Outside the auxiliary strip, no selected pupil work is erased during the lesson.

Time	Board state	What is written	Speech and activity	Board action / status
Before entry	T0 - discreet outline	Draw three zones; reserve P1–P4 and the right-hand side.	Prepare the seating plan; do not pre-write the answer.	RESERVE spaces; everything remains available.
0–10	T1 - Phase 1 (0–10): Fluency + review + “check without calculating”	Go over 3.3 and the brevet question 6.3 (French national exam at the end of Year 10), then tackle 5.2 without writing down calculations: $3^2 = 9$, a reduction shortens, the largest angle faces the longest side and an area follows k^2 .	“Which sense, dimension or order-of-magnitude check is enough to rule out this result?”	ERASE calculations; keep four plausibility criteria.
10–25	T2 - Phase 2 (10–25): Problem posing from 5.3 (the peak)	Change $OC : 4 \rightarrow 3$ and predict the effect on k , OD and AB before calculating. Have pupils create two similar triangles with scale factor $\frac{3}{4}$, swap problems, solve them and revise them.	“If k decreases, which of the two lengths must decrease and which must increase?”	KEEP the model, two contrasting problems and their revised version.
25–28	T3 - Phase 3 (25–28): Presenting the Problem Wheel	Write the four quadrants: what I know, what I want, my tools, my check. Link them explicitly to problem 4.1.	“Which question on the Wheel must be answered before allowing the first calculation?”	KEEP the Wheel in the right-hand zone as a reusable tool.
28–40	T4 - Phase 4 (28–40): 3.5 the Statue of Liberty (critical thinking, pairs)	a) quick whole-class step: $k = \frac{11,5}{46} = \frac{1}{4}$. b) in pairs, 8 min.	“If the replica were a perfect scale reduction, what would it weigh? Which quantity does mass follow?”	REMAINS: selected work, anonymous code C and written connector.
40–52	T5 - Phase 5 (40–52): 4.1 the scale model park, using the Wheel	Work through all 4 quadrants before any calculation; sketch required (question a).	“What I know? What I want? My tools? My check?”	KEEP only instructions and time marker; do not guide.
52–55	T6 - Phase 6 (52–55): Handing out worked solutions + announcing end of R	Hand out the worked solutions and state the four rules: try and write first, open afterwards, compare your method, then explain your error.	“When do worked solutions become a learning tool rather than a substitute for thinking?”	FINAL PHOTO: keep the usage agreement in the right-hand zone.

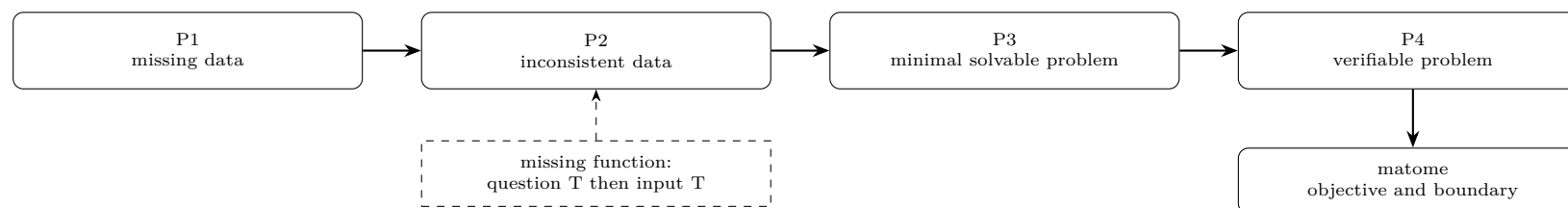
Time consistency check: copying only starts after comparison; if time runs short, 4.1 e) moves to the opening starter of S6 and revising the problems can be finished in S6. Cannot be cut: predicted modification, creation, revising a problem against the four criteria, critical thinking on 3.5 and presenting the Wheel.

3. Managing the discussion: select, sequence, connect

ID	Anticipated pupil work	Role in the narrative	Exact questions	Placement and status
P1	A problem omits the vertex correspondence: the matching has to be guessed.	Make it visible that a good problem gives the necessary information unambiguously.	"Can we match the sides without asking the author? What information is missing?"	Centre, block 1; annotate "missing correspondence", then revise.
P2	A problem adds a third length that contradicts the scale factor $\frac{3}{4}$.	Have pupils check the internal consistency of the data they created.	"Can all three stated ratios be equal to $\frac{3}{4}$?"	Centre, block 2; cross out the inconsistent value and correct it.
P3	Minimal solvable problem: explicit correspondence, two corresponding lengths fixing $k = \frac{3}{4}$ and one unknown in each triangle.	Bring out the sufficient conditions for a similarity problem.	"Is this problem solvable as it stands? What is necessary, redundant or missing?"	Centre, block 3; keep the revised version.
P4	Complete problem that makes it possible to check all three ratios and large/small consistency.	Add an explicit check to the solution, not just data.	"What check does the problem make possible after calculating?"	Centre, block 4; keep only if it provides this check.

Planned selection order. Initial answer or productive error → comparison → stabilised procedure → justification or matome. After each contribution: "Who used this idea?" then "Who can suggest a more general justification?"

Essential / contingent. The functions of P1–P4 are essential; only the exact author and the wording used are contingent. If a function is missing from the pupil work, ask a public question T and then sign any teacher input with T. On the actual board, write C1–C4 and later fill in the C/P correspondence.



Structured variation. V1 modify a single given value in problem 5.3 and predict its effect; V2 create a problem where the second triangle is a reduction of the first with scale factor $\frac{3}{4}$; V3 have peers solve and then revise the problem against four criteria.

Invariant to elicit. A good problem is solvable, provides sufficient and consistent data, states the vertex matching explicitly and allows a check. The Wheel separates data, goal, tools and check.

Scope of validity. A scale $\frac{1}{n}$ multiplies lengths by $\frac{1}{n}$, areas by $\frac{1}{n^2}$ and volumes by $\frac{1}{n^3}$; a calculation is only relevant if the modelled quantity is identified.

4. Comparing the plan with the board actually built

Observed item	Planned	Actual / evidence collected
Exploration and whole-class discussion time	0–10 check without calculating; 10–25 modify, create and revise; 25–28 Wheel; 28–40 statue; 40–52 park; 52–55 worked-solutions agreement.	N/A
Pupil work that emerged	P1 missing correspondence; P2 inconsistent data; P3 minimal solvable problem; P4 problem with a built-in check.	N/A
Order of presentation	initial / conflict → comparison → method → justification	N/A
Useful unexpected work	another valid and verifiable problem form, if it appears	N/A
Question that moved thinking forward	“How do we know a result is wrong without recalculating it?”; “Before any calculation: if OC goes from 4 to 3, what happens to k , OD and AB ?”	N/A
Elements left visible	problem, C1–C4, connections, invariant, matome	N/A
Elements erased and when	auxiliary calculations only, after validation and verbalisation	N/A
Matome stated by the pupils	Creating a problem forces you to check the sufficiency, consistency and verifiability of the data; the Wheel structures modelling and critical thinking.	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert the photograph here
or state its file name

Photograph check

- ☐ The problem and the goal are identifiable.
- ☐ C1–C4 are distinguishable and linked to the P functions.
- ☐ A refuted error is not confused with the result.
- ☐ Relationships and connectors are written.
- ☐ The invariant or condition is visible.
- ☐ The matome makes sense without the teacher's spoken commentary.

Class indicator

Observed work or change of mind: _____

Delayed copying

Actual start time: _____

Local decision

Element to keep: _____

Single adjustment

Element to modify: _____

Bansho keikaku - planned board

Lesson question. Program two similar triangles, distinguish interior angle and turn angle, justify the scale factors for lengths and areas; apply to a question from the brevet (French national exam at the end of Year 10).

LEFT: question and data

How can we draw two triangles of the same shape by changing one variable?

Sides: 60 then 90 steps. Same loop and same turns.

CENTRE: trials and comparison

Loop: repeat 3 times, move `cote` steps, turn right 120° .

Interior angle: $180^\circ - 120^\circ = 60^\circ$.

Each sprite draws three sides of equal length. Pen up to change starting point.

RIGHT: justified summary

Angles equal in pairs: similar triangles.

$k = 90/60 = 1,5$.

Perimeters: 180 and 270 steps.

Areas: factor $k^2 = 2,25$.

With 30 steps: $k = 1/2$, area divided by 4.

Delivery. Reveal the right-hand column only after comparing actual pupil work. A missing error becomes a teacher question, never a fabricated script.

0–10 min: Review of 4.2. Compare the scale model and reality: $540,8 \times 125^2 = 8\,450\,000 \text{ cm}^2 = 845 \text{ m}^2$. Correct an actual dimension error. Do not add a second problem.

10–14 min: Predict before running. Exercise sheet 3.8 a): have pupils complete the loop individually. Ask why 60° or 120° , without letting the software settle it. Gather justifications.

14–30 min: Program and run. 3.8 b)–c): first pupil at the keyboard for the side of 60, second for the side of 90. Check that the figure closes, the pen is lifted between figures and the initial direction is the same. Each pupil keeps the program or screenshot and records their contribution.

30–40 min: Compare and justify. 3.8 d)–e): $k = 1,5$, perimeters 180 and 270, area ratio 2,25. Require equal angles to justify similarity. With 30 steps: $k = 1/2$, perimeter 90, area divided by 4. Confront prediction and result; a screenshot does not replace these reasons.

40–50 min: Brevet transfer. Exercise sheet 6.4 written individually: angles, correspondence, ratios, unit and check. Mark only the key steps.

50–55 min: Matome and test. Have pupils state the scaling rule, then announce the test: recognise, justify, calculate, k , k^2 , k^3 effects, common proof. Hand out the Memo. Problem 3.6 on cones remains available in bank R; it is not added to this lesson.

Evidence to observe. Initial answer, executed procedure or construction, final justification and exit ticket. Note which pupils need follow-up and the time actually used.

After the lesson: observed success: _____

Follow-up needed: _____

Time adjustment: _____

Bansho keikaku - planned board

Chapter 01A - S7 - Similar triangles

1. Before the lesson: intention and planned final board

Type: test and delayed feedback

Kadai	Where is the first breakdown in this paper, how do we fix it, and what rule will prevent the mistake from happening again?	Objective	Assess without implicit help the mastery of: recognising similar triangles, matching corresponding sides, calculating lengths, angle characterisation and effects on areas and volumes. After marking, locate a real mistake, fix it and formulate a transferable prevention rule.
Equipment	Checked test paper and mark scheme; sheets, permitted equipment and timer. For feedback: scripts, code grid, anonymised work and error log.	Pupil work	P1–P4 planned and essential: intact script, located breakdown, minimal fix, prevention rule.
ESS essential function CONT contingent form DECL work to prompt. Actual pupil work is labelled C1, C2, C3, C4 with no name or initial; C is then linked to functions P.			

Planned board for returning scripts: do not display during the test.

IDEAL BOARD (final state): to project or reproduce

LEFT: the real mistake

Kadai: Where is the first breakdown in this paper, how do we fix it, and what rule will prevent the mistake from happening again?

[copy here, as is and anonymised, the most frequent mistake from the scripts: only one]

Dominant codes from the test:

R ____ A ____ K ____ D ____ U ____

CENTRE: whole-class treatment

“Is this first line right? Why?”: rephrased by two pupils, never by the teacher

Generalisation, written: “in which family of figures could this mistake come back?”

1.5e vs Ex. 5 replayed side by side: 80° (two positions) against 100° (obtuse $\rightarrow$ only one)

Follow-up: the dominant codes feed into the fluency skills of chapter 01B

RIGHT: error log

The entry format, displayed:

1. the problem in one line
2. MY mistake
3. the full correction
4. “I made a mistake because...”

Reread before each test.

Material rule: problem, selected work, refuted mistakes, relationships and matome remain visible until the final photograph. Only auxiliary calculations may be erased after validation and verbalisation.

2. Storyboard: write, show, link, keep, erase

Stable code: main writing in base colour; relationship, logical connector or invariant in an accent colour; mistake crossed out but legible. Outside the auxiliary strip, no selected work is erased during the lesson.

Time	Board state	What is written	Speech and activity	Material action / fate
Before entry	T0 - neutral board	Write only: “50 min”, calculator allowed, permitted equipment and end time. Prepare no visible correction area.	Place the test papers face down; check that the board provides no content help.	DISPLAY NOTHING from the feedback board or the P functions.
0–2	T1 - Settling in and reading	Remind pupils of the duration, permitted equipment, mark scheme and handing-in rule; have them look through the paper before starting.	Which instructions must you check before starting?	REMAINS: neutral instructions and time markers only.
2–52	T2 - Individual test	Show no method or result; note the items that block many pupils and the scripts finished very early.	No content questions; rephrase an instruction without guiding the method.	ADD NOTHING that could constitute implicit help.
52–55	T3 - Checking and collection	Announce the remaining time, have pupils check name, pages and units, then collect without public correction.	Which item must you reread before handing in, without asking for validation?	ERASE the countdown after collection; keep the teacher’s observations out of sight.
delayed feedback	T4 - Clinic on a real mistake	Before entry, prepare the three areas and copy an anonymised piece of work. In class, locate the first breakdown, compare the fix and generalise a prevention rule.	At which line does the reasoning cease to be justified, and what minimal change fixes it?	FINAL PHOTO: legible mistake, linked correction, dominant code and prevention rule.

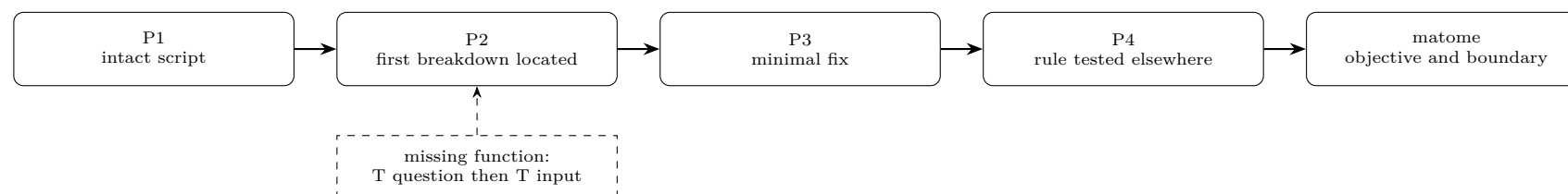
Time consistency check: the test keeps a full 50 minutes, from 2 to 52 minutes; settling in and collection share the remaining 5 minutes. Returning scripts belongs to the next slot: a single real mistake is treated from the intact script to the prevention rule, without an exhaustive correction on the board.

3. Producing the discussion: select, order, connect

ID	Anticipated pupil work	Function in the narrative	Exact questions	Placement and fate
P1	A real mistake, anonymised and copied without correction.	Separate what is actually written from what we assume the pupil meant to do.	What exactly does this script say, without judging it yet?	Left; keep intact until comparison.
P2	The localised diagnosis: first unjustified line and associated error code.	Make the precise point of breakdown visible rather than correcting the whole script.	At which line does the reasoning cease to be justified?	Centre left; link explicitly to P1.
P3	A minimal, complete and verifiable fix of the faulty line.	Compare before and after, then restore the missing property, method or condition.	What minimal change makes this answer valid?	Centre right; keep with the corrected connector.
P4	A transferable prevention rule and a neighbouring example where it applies.	Generalise without reducing feedback to the correction of a single exercise.	In what other family of exercises could this mistake come back? How can we check that the proposed rule works there?	Right, before the matome; keep a checkable formulation.

Planned selection order. Initial answer or productive mistake → comparison → stabilised procedure → justification or matome. After each contribution: “Who used this idea?” then “Who can suggest a more general justification?”

Essential / contingent. The functions of P1–P4 are essential: without a prevention rule or a neighbouring example, the Kadai remains unanswered. The exact author and wording remain contingent. On the actual board, write C1–C4 and then fill in the C/P correspondence.



Organised variation. V1 revisit the isosceles triangle with 80° , which allows two placements; V2 replace with 100° , which cannot be at the base; V3 formulate the rule on the sum of angles.

Invariant to be verbalised. During the test, no content help is added. During feedback, the first unjustified line is located before any fix; the correction restores an explicit and checkable property.

Boundary of use. The error code describes the visible breakdown, not the pupil's assumed intention. One real mistake is enough to generalise; feedback is not an exhaustive correction of the test paper.

4. Comparing the plan and the board actually built

Observed point	Planned	Actual / evidence gathered
Exploration and whole-class discussion time	0-2 settling in; 2-52 individual test; 52-55 checking and collection; delayed feedback: clinic on a real mistake	N/A
Pupil work that appeared	P1 intact anonymised script; P2 first break-down and code; P3 minimal fix; P4 prevention rule and neighbouring example.	N/A
Order of presentation	initial / conflict → comparison → method → justification	N/A
Useful unexpected work	neighbouring example different from the anticipated one, integrated into P4	N/A
Question that moved things forward	“What exactly does this script say?”; “At which line does the reasoning cease to be justified?”	N/A
Elements kept visible	problem, C1-C4, relationships, invariant, matome	N/A
Erased elements and timing	time markers of the test after collection; during feedback, auxiliary calculations only after validation	N/A
Matome formulated by the pupils	A useful correction names the first break-down, fixes it using a property and formulates a check applicable to a neighbouring exercise.	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert the photograph here
or indicate its filename

Photograph check

- ☐ The problem and the goal are identifiable.
- ☐ C1-C4 are distinguishable and linked to the P functions.
- ☐ A refuted mistake is not confused with the result.
- ☐ The relationships and connectors are written.
- ☐ The invariant or condition is visible.
- ☐ The matome can be understood without the teacher’s spoken commentary.

Class indicator	Observed work or change of mind: _____	Delayed copying	Actual start time: _____
Local decision	Element to keep: _____	Single adjustment	Element to modify: _____