



Exercise sheet

Similar triangles: Enlargements and reductions

Exercises from the core and recommended tracks (numbering from the full sheet)

Difficulty: ★☆☆☆ very easy ★★☆☆ easy ★★★☆ medium ★★★★★ hard

Track: 🏠 core 🎯 recommended 🏰 going further

Core framework. Two triangles are similar when the lengths of their sides are proportional; the constant k is the scale factor. Matching sides are called corresponding sides.

Angle criterion. Two triangles are similar if, and only if, their angles are equal in pairs, and two pairs of equal angles are enough (the third is 180° minus the other two).

Where to find equal angles? Marked information given in the problem • shared angle • vertically opposite angles • alternate interior or corresponding angles formed with parallel lines • calculating the third angle.

Enlargement–reduction with scale factor k : lengths and perimeters are multiplied by k , angles keep their size, areas are multiplied by k^2 , volumes by k^3 . A scale of $\frac{1}{n}$ is a reduction with scale factor $k = \frac{1}{n}$.

Working backwards: if the problem gives the areas, their ratio is k^2 ; we find k by looking for the positive number whose square is equal to this ratio.

Common pitfalls:

- Do not confuse “similar” and “congruent”: two similar triangles have the same shape, not necessarily the same size (two congruent triangles are similar with $k = 1$).
- The scale factor is read between corresponding sides (paired using equal angles), never between sides chosen at random.
- Areas are multiplied by k^2 , never by k ; volumes by k^3 .
- A single pair of equal angles is not enough; you need two pairs of equal angles.
- Watch the direction of k : multiply by k in the direction used to calculate it, divide by k in the other; check that your answer makes sense (the smaller triangle must have the shorter sides).

Basic method, the three steps:

1. Pair up: identify the corresponding vertices using equal angles (opposite equal angles lie the corresponding sides).
2. Set up: a proportionality table of the corresponding sides, one triangle per row.
3. Solve: calculate k from a complete column, then multiply or divide by k (or cross-multiply). Check: are all three ratios equal? Is the longest side opposite the largest angle?

Fluency (no calculator).

- | | |
|---|---|
| a) Two angles of a triangle: 47° and 86° . The third? | g) Simplify $\frac{3}{4,5}$. |
| b) Two angles of a triangle: 90° and 26° . The third? | h) Reduction with scale factor 0,5: a length of 7 cm becomes... |
| c) Angles 40° and 60° here, 60° and 80° there: similar? | i) $k = 1,2$ and perimeter 15 cm: new perimeter? |
| d) Enlargement with scale factor 3: the area is multiplied by... | j) Scale $\frac{1}{100}$: 3 cm on the plan represent... |
| e) Enlargement with scale factor 2: the volume is multiplied by... | |
| f) The area was multiplied by 25: what is k ? | |

Retrieval practice (previous chapters: 5 questions).

1. (Ch. 00) Write 360 as a product of prime factors.
2. (Ch. 01) Write $\frac{54}{36}$ in lowest terms.
3. (Ch. 01) Calculate $\frac{3}{5} \times \frac{10}{9}$.
4. (Ch. 01) Calculate $(0,5)^3$ and $\left(\frac{2}{3}\right)^2$.
5. (Ch. 01) Write $1,25 \times 10^4$ as an ordinary decimal number.

1. Identifying similar triangles

1.1. Similar or not? (using angles)

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In this set of questions, we compare a triangle ABC and a triangle DEF .

In each case, state whether the two triangles are similar. Justify your answer by calculating the third angle when necessary.

- In ABC , $\hat{A} = 47^\circ$ and $\hat{B} = 86^\circ$. In DEF , $\hat{D} = 47^\circ$ and $\hat{E} = 47^\circ$.
- In ABC , $\hat{A} = 35^\circ$ and $\hat{B} = 72^\circ$. In DEF , $\hat{D} = 73^\circ$ and $\hat{E} = 35^\circ$.
- In ABC , $\hat{A} = 50^\circ$ and $\hat{B} = 60^\circ$. In DEF , $\hat{D} = 60^\circ$ and $\hat{E} = 65^\circ$.
- ABC is right-angled at A with $\hat{B} = 32^\circ$. DEF is right-angled at D with $\hat{E} = 58^\circ$.
- In ABC , $\hat{A} = 44^\circ$ and $\hat{B} = 68^\circ$. DEF is isosceles at D with $\hat{D} = 92^\circ$.
- ABC is equilateral. In DEF , $\hat{D} = 60^\circ$ and $\hat{E} = 60^\circ$.

1.2. Similar or not? (using sides)

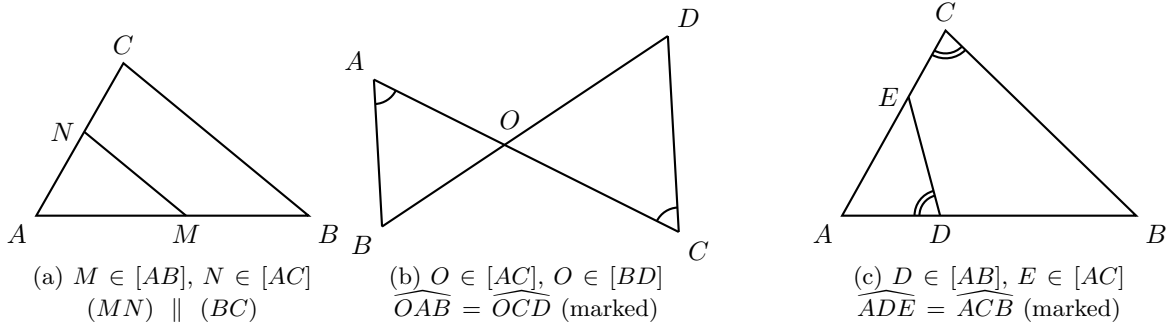
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- In each case, are triangles ABC and DEF similar? Justify using all three ratios.
 - ABC ($AB = 3$; $BC = 4$; $AC = 5$) and DEF ($DE = 6$; $EF = 8$; $DF = 10$).
 - ABC ($AB = 3$; $BC = 4$; $AC = 5$) and DEF ($DE = 4$; $EF = 5$; $DF = 6$).
 - ABC ($AB = 4,5$; $BC = 6$; $AC = 7,5$) and DEF ($DE = 3$; $EF = 4$; $DF = 5$).
- Critical thinking. ABC and EFG are two triangles such that: $AB = 5$ cm, $AC = 8$ cm and $BC = 6,5$ cm; $EF = 1$ cm, $EG = 1,6$ cm and $FG = 1,2$ cm.
 - Is triangle EFG a reduction of triangle ABC ?
 - If not, change one length so that it is. State the scale factor of the reduction.

1.3. Justifying similarity: three configurations

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For each figure, prove that the two given triangles are similar, using the standard model proof from the course notes (two pairs of equal angles, each justified).

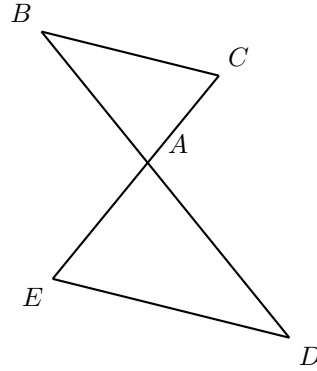


- Figure (a): prove that triangles AMN and ABC are similar.
- Figure (b): prove that triangles OAB and OCD are similar.
- Figure (c): prove that triangles ADE and ACB are similar, then state the three pairs of corresponding vertices. Careful: here, does E correspond to C ... or to B ? Let the angles guide you, not the position.

1.4. The bowtie configuration

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The line segments $[BD]$ and $[CE]$ intersect internally at A , and the lines (BC) and (DE) are parallel. We are given $AB = 3,6$ cm and $AD = 4,8$ cm.



- What can be said about triangle ADE in relation to triangle ABC ? Justify using angles (no intercept theorem (Thales' theorem in France) here!), and state the scale factor from ABC to ADE .
- Find AE given that $AC = 2,4$ cm.
- Find BC given that $DE = 5,2$ cm.

1.5. True or false?

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For each statement: true or false? A “true” statement must be proved (using angles); a “false” statement must be disproved with a numerical counterexample.

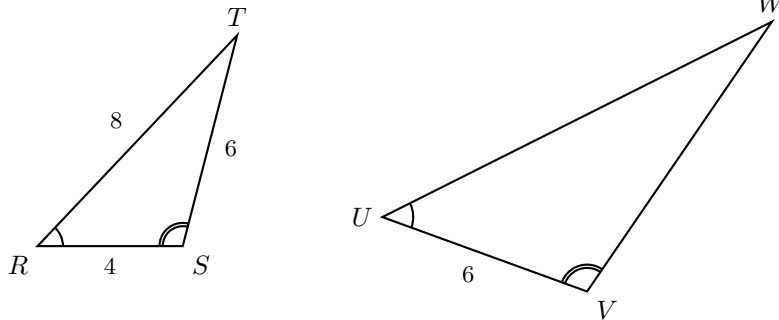
- Two equilateral triangles are always similar.
- Two isosceles triangles are always similar.
- Two right-angled triangles are always similar.
- Two isosceles right-angled triangles are always similar.
- Two isosceles triangles each having an 80° angle are always similar.
- Two similar triangles are always congruent.
- Two congruent triangles are always similar.

2. Calculating lengths

2.1. First calculation, two ways

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Triangles RST and UVW are similar, with the correspondence $R \leftrightarrow U$, $S \leftrightarrow V$, $T \leftrightarrow W$ (angles marked on the figure). We are given: $RS = 4$; $ST = 6$; $RT = 8$; $UV = 6$.



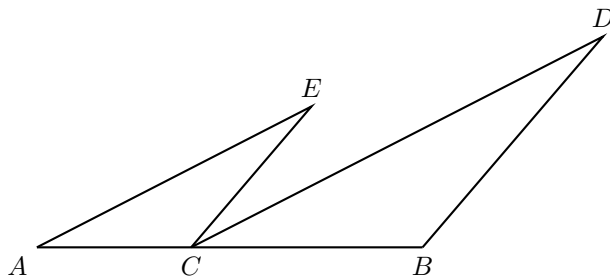
- Give the scale factor k from RST to UVW .
- Calculate VW in two ways: using the scale factor k , then by cross-multiplying. Which of the two is faster here, and why?
- Calculate UW .
- Calculate the perimeter of each triangle. What do you notice? Was this predictable?

2.2. The brevet (French national exam at the end of Year 10) problem

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This is the classic problem of the chapter: you should be able to do it with your eyes closed.

In the figure below: point C lies on the line segment $[AB]$; $AC = 3$; $AB = 7,5$; $BD = 5,4$ and $CD = 9$; the lines (AE) and (CD) are parallel; the lines (CE) and (BD) are parallel.

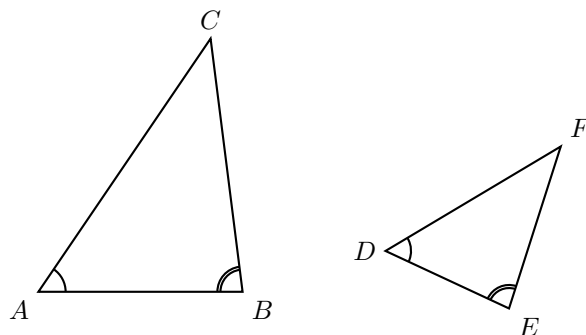


- Prove that the angles $\widehat{BCD}$ and $\widehat{CAE}$ are equal.
- Prove that triangles ACE and CBD are similar.
- Deduce the lengths of the sides of triangle ACE .

2.3. Corresponding sides: two configurations

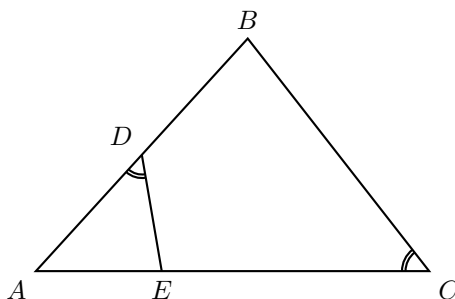
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Configuration 1. Triangles ABC and DEF below have their angles marked: $\widehat{A} = \widehat{D}$ and $\widehat{B} = \widehat{E}$. We are given: $AB = 6$; $BC = 7,5$; $AC = 9$; $DE = 4$.



- Justify that triangles ABC and DEF are similar.
- Draw and complete a table of corresponding sides, then give the scale factor from ABC to DEF .
- Calculate EF and DF .

Configuration 2. In triangle ABC , D is a point on $[AB]$ and E is a point on $[AC]$ such that $\widehat{ADE} = \widehat{ACB}$ (marked on the figure). We are given: $AD = 4$; $AC = 10$; $AB = 8$; $CB = 7,5$.



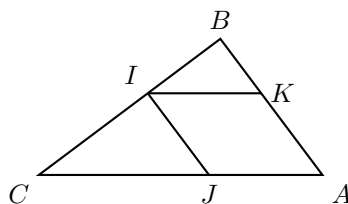
- Prove that triangles ADE and ACB are similar.
- State the corresponding sides. Trap ahead: $[AD]$ is not compared with $[AB]$...
- Calculate DE and AE .

2.4. The 3-4-5 triangle

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(Unit of your choice.) Let ABC be a triangle such that $AB = 3$, $BC = 4$ and $CA = 5$.

- Prove that ABC is right-angled at B .
- Let $J \in [CA]$ be such that $CJ = 3$. The line parallel to (AB) passing through J intersects (BC) at I ; the line parallel to (CA) passing through I intersects (AB) at K . Copy the figure below at full scale and complete it.



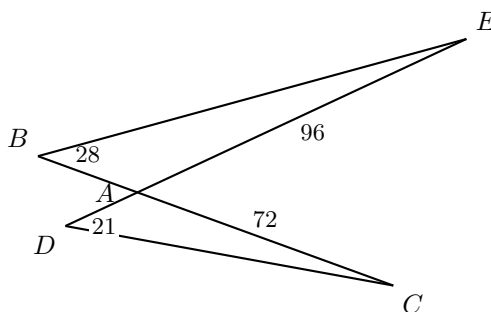
- Prove that triangles JIC and ABC are similar (using angles), give the scale factor from ABC to JIC , then calculate CI and JI .
- Deduce BI , then prove that triangles KBI and ABC are similar and calculate KB and KI .
- Calculate the area of triangle ABC , then, without calculating any further lengths, the areas of triangles KBI and JIC (think of k^2). Deduce the area of quadrilateral $AKIJ$.

2.5. The bowtie with proportional sides

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SAS extension from section 2.3 of the course notes: redo the example without looking at the notes, then take it further.

The line segments $[BC]$ and $[DE]$ intersect internally at A . We are given: $AB = 28$ mm; $AC = 72$ mm; $AD = 21$ mm; $AE = 96$ mm. Careful: no pair of parallel lines is given.



- Prove that triangles DAC and BAE are similar. Which criterion from the course notes applies here, and why was the angle criterion alone not enough?
- What is the ratio of the areas, from DAC to BAE ? State which of the two has the larger area.
- In exercise 1.4, the “wings” of the bowtie were parallel. Are the lines (DC) and (BE) parallel here? Justify. (Hint: if they were, which alternate interior angles would be equal for the transversal (BC) ? Compare with what the similarity in a) tells us.)

3. Enlargements and reductions

3.1. Drills: the effects of k , k^2 , k^3

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No calculator: almost everything can be done mentally.

- $AB = 12$ cm, $AC = 8$ cm, $BC = 6$ cm, and triangle AMN is a reduction of triangle ABC with scale factor 0,25. Mentally calculate the perimeter of triangle AMN .
- The area of a triangle is 25 cm^2 . It is enlarged with scale factor 2: new area?
- A cube has a volume of $27\,000 \text{ m}^3$. A scale model is made at a scale of $\frac{1}{30}$: volume of the model?
- The area of a shape was multiplied by 9: what was the scale factor of the enlargement?
- The lengths of a solid are multiplied by 5: its volume is multiplied by...
- Multiple choice. A camera screen is a rectangle of length 5,4 cm; the printed photo has a length of 15 cm. The scale factor of the enlargement is: A. 2,7 B. 2,8 C. $\frac{25}{9}$
- Multiple choice. A reduction can for example have a scale factor of: A. 120 % B. $\frac{4}{3}$ C. $\frac{3}{4}$
- In an enlargement with scale factor k , the sizes of the angles are multiplied by... (careful!)

Application 3.1b: the photographed pennant

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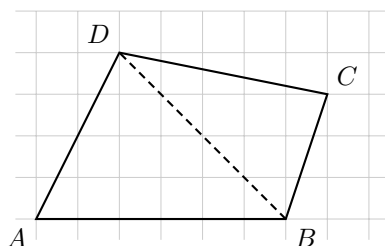
In a photograph, a triangular pennant has one side of 4 cm and another of 3 cm. The real pennant, which is similar to the one in the photograph, has a length of 60 cm for the side corresponding to the 4 cm side.

- What is the scale factor of the enlargement from the photograph to the real pennant?
- What is the real length of the side corresponding to the 3 cm side? Check that your answer makes sense.

3.2. The quadrilateral: conjecturing with k and k^2

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$ABCD$ is the quadrilateral below, drawn on a 1 cm square grid; the diagonal $[BD]$ has been drawn.



- On a grid, draw a reduction $A'B'C'D'$ of this quadrilateral with scale factor 0,5.
- How does the size of angle $\widehat{D'A'B'}$ compare with that of $\widehat{DAB}$? (No protractor needed.)
- Calculate the area of triangle ABD , then that of triangle $A'B'D'$. Copy and complete: $\mathcal{A}' = 0,5 \cdots \times \mathcal{A}$.
- Without drawing it, give the area of triangle $A''B''D''$ for an enlargement $A''B''C''D''$ with scale factor 1,2.
- Copy and complete the conjecture: "In an enlargement or a reduction with scale factor k , lengths are..., angles are..., areas are..."

3.3. Working backwards: finding k

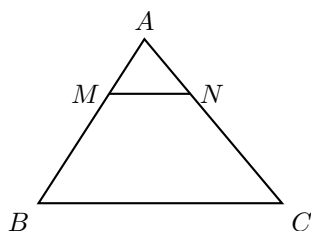
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- Reasoning. A' and B' are points on the rays $[CA)$ and $[CB)$ such that (AB) and $(A'B')$ are parallel, with $AB = 1,5$ cm and $A'B' = 2$ cm. The area of triangle ABC is $2,7 \text{ cm}^2$. Prove that triangles CAB and $CA'B'$ are similar (using angles), then calculate the area of triangle $CA'B'$.
- The area of a shape is 72 cm^2 . After an enlargement, the resulting shape has an area of $1\,800 \text{ cm}^2$. What is the scale factor of this enlargement?
- Multiple choice. A rectangle $A'B'C'D'$ of area 24 cm^2 is the enlargement with scale factor 1,25 of a rectangle $ABCD$. The area (in cm^2) of $ABCD$ is: A. 15,36 B. 19,2 C. 30 D. 37,5
- On a plan at a scale of $\frac{1}{3\,000}$, a football pitch has an area of 8 cm^2 . What is the real area (in m^2) of this pitch? Is the result plausible for a football pitch?

3.4. Is Arthur right? 🎯

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We know that: points A, M, B are collinear in this order, as are A, N, C ; the lines (MN) and (BC) are parallel; $AB = 6$ cm and $AM = 2$ cm; the area of triangle ABC is 9 cm^2 .



- Arthur claims: “Triangle AMN is a reduction of triangle ABC ”. Is he right? Explain, and if so give the scale factor of the reduction.
- Deduce the area of triangle AMN .

3.5. The Statue of Liberty: critical thinking 🎯

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The Statue of Liberty in New York, with a height (excluding the pedestal) of 46 m, was designed by the French sculptor Auguste Bartholdi (1834–1904). A replica stands on the Île aux Cygnes in Paris; its height is 11,50 m.

- What is the scale factor of the reduction from the New York statue to the Paris statue?
- Critical thinking. The mass of a statue is linked to the volume of the materials used. To test a simplified model inspired by these statues, we use masses of 225 tonnes and 14 tonnes here; these are the conventional data for this exercise. Are these masses consistent with a reduction of scale factor $1/4$ in all dimensions, including thicknesses, using the same materials in the same places? Justify by a calculation. Can we deduce from this that their outer shapes are not similar?

3.6. Ice cream cones 🎯

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A company sells three sizes of ice cream cones of the same shape: large, with a volume of 720 mL, medium and small.

- The medium cone is a reduction of the large cone with scale factor 75 %. Calculate its volume.
- The small cone is a reduction of the large cone with scale factor 50 %. Marie claims: “The volume of the small cone is equal to half that of the medium cone.” Is she right? Justify with a full calculation.
- Summary question: by what percentage does the volume decrease when the lengths decrease by 25 %? State the takeaway of this exercise in one sentence.

3.8. Scratch: two triangles of the same shape 🐼

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Add the Pen extension and create the variable `cote`. The unit of length is the Scratch step.

- Predict. Fill in the two missing numbers in the script to draw an equilateral triangle of side 60 steps.

When green flag clicked:

- pen up; erase all; go to $x = -180, y = 60$; point in direction 90° ;
- set `cote` to 60; pen down;
- repeat times:
 - move `cote` steps; turn right degrees;
- pen up.

- Program and run. Build the completed script, then run it. Check that the triangle closes. Explain why the turning angle is not 60° .
- Enlarge. Continuing the script, without erasing the first drawing: go to $x = 20, y = 60$, point in direction 90° , set `cote` to 90, pen down, repeat the same loop, then pen up. Predict the scale factor of the enlargement, then run the script.
- Justify. Why are the two triangles similar? Calculate both perimeters. By what factor is the area multiplied from the first triangle to the second? Keep the program and a screenshot of the two triangles, together with these justifications.
- Apply. Replace 90 with 30. Predict the scale factors for the lengths, angles, perimeter and area of the second triangle compared with the first; check by running the script.

4. Problems

4.1. The triangular park to scale

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A triangular playground DEF is drawn to scale. Actual dimensions: $DE = 12$ m, $EF = 9$ m, $DF = 15$ m. On the drawing, $DE = 6$ cm.

- Before any calculation: make a freehand sketch showing the actual data and that of the drawing.
- What is the scale of the drawing?
- Give the lengths EF and DF on the drawing.
- Prove that the actual park is right-angled at E , then calculate its actual area.
- Is the area of the drawn triangle equal to the actual area divided by 200? Justify without calculating the area of the drawing, then check by calculation.

4.2. The 1/125 scale model plane

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A scale model of a jumbo airliner is made at a scale of $\frac{1}{125}$.

- The length of the plane is 73 m. What is that of the model? Give the result in centimetres.
- The wing area of the model is $540,8 \text{ cm}^2$. What is the wing area (in m^2) of the plane?
- The fuel tank of the plane holds 310 000 L. What is the capacity (in cm^3) of that of the model?
- Is each result plausible? (One sentence per question.)

5. Justifying and reasoning

5.1. Spotting the mistake: Naïm's work

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Triangles ABC and DEF are similar, with $\hat{A} = \hat{D}$ and $\hat{B} = \hat{E}$. We are given $AB = 4$, $BC = 6$ and $DE = 6$. We need to calculate EF .

Naïm's work. " $BC = 6$ and $DE = 6$, so the scale factor is $\frac{6}{6} = 1$: the triangles are congruent, and $EF = BC = 6$."

- Naïm's result is wrong. Identify the exact mistake made (which step of the method was skipped?).
- Write out the correct solution and give the true value of EF .

5.2. Checking consistency: without calculating

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Each of the following statements is false, and this can be seen without writing down a single calculation. Explain why, in one sentence each time.

- "An enlargement with scale factor 3 multiplied the area of the figure by 6."
- "In my triangle reduced at a scale of $\frac{1}{2}$, the 8 cm side became a 16 cm side."
- "These two triangles are similar: in the first one, the longest side is opposite the largest angle; in the second one, which is not equilateral, the longest side is opposite the smallest angle."
- "The model is at a scale of $\frac{1}{100}$, so the area of paint needed is 100 times smaller than for the actual object."

5.3. Writing a complete solution

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Line segments $[AC]$ and $[BD]$ intersect at O . We know that $\widehat{OAB} = \widehat{OCD}$, and we are given: $OA = 6$; $OC = 4$; $OB = 5,4$; $CD = 5$. Write a complete, self-contained solution following the template practised in the chapter:

- prove that triangles OAB and OCD are similar (two pairs of equal angles, each justified);
- match the vertices, set up the table, then calculate OD and AB ;
- check (equal ratios, large/small consistency) and conclude with a sentence.

Modify, create, have someone solve

1. Modify and predict. In the previous problem, replace $OC = 4$ with $OC = 3$, without changing OA , OB or CD . Before calculating, predict the effect on the scale factor from OAB to OCD , on OD and on AB , then check.
2. Create under constraints. Invent a problem about two similar triangles, the second being a reduction of the first with scale factor $\frac{3}{4}$, with one unknown length in each triangle and enough data to calculate them. Specify the correspondence of the vertices unambiguously.
3. Solve, check, swap. Solve the problem by justifying the similarity and the matching, check the three ratios, then have a classmate solve the problem. Revise any unnecessary, missing or ambiguous data.

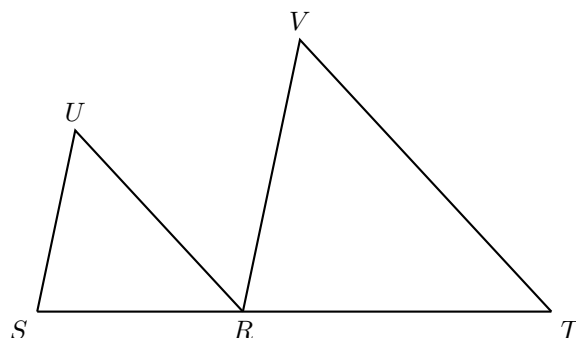
6. Brevet-style problems (French national exam at the end of Year 10)

Problems 1 to 6 are at  level. Problems 1 and 2 deliberately reuse the structure of exercise 2.2: this is practice for the exam format.

6.1. Brevet 1: Similar triangles on a line

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In the figure below: point R lies on the line segment $[ST]$; $SR = 4$; $ST = 10$; $RV = 5,4$ and $TV = 7,2$; the lines (SU) and (RV) are parallel; the lines (RU) and (TV) are parallel.

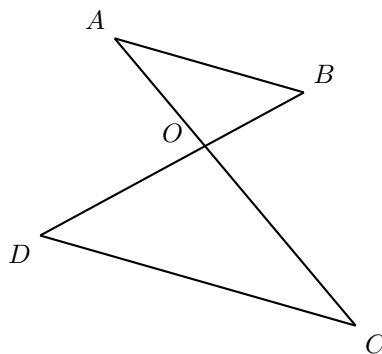


- Prove that the angles $\widehat{USR}$ and $\widehat{VRT}$ have the same measure.
- Prove that triangles SRU and RTV are similar.
- Hence find the lengths SU and RU .

6.2. Brevet 2: The butterfly

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Line segments $[AC]$ and $[BD]$ intersect at O , and the lines (AB) and (CD) are parallel. We are given: $OA = 3$; $OB = 2,4$; $AB = 4,2$; $OC = 5$.



- Prove that triangles OAB and OCD are similar.
- Calculate CD and OD .

6.3. Brevet 3: Similar triangles and areas

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Triangles ABC and DEF are similar, with $\hat{A} = \hat{D}$ and $\hat{B} = \hat{E}$. We are given: $AB = 6$ cm; $DE = 4$ cm; the area of triangle ABC is $13,5$ cm²; the perimeter of triangle ABC is 18 cm.

- Give the scale factor from ABC to DEF .
- Calculate the perimeter of triangle DEF .
- Calculate the area of triangle DEF .

6.4. Brevet 4: Recognise, then calculate

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Triangle JKL satisfies $\hat{J} = 35^\circ$ and $\hat{K} = 60^\circ$. Triangle MNP satisfies $\hat{M} = 85^\circ$ and $\hat{N} = 60^\circ$.

- Prove that these two triangles are similar.
- Specify the three pairs of corresponding vertices.
- We are given $JK = 8$ cm, $KL \approx 4,6$ cm (rounded to the nearest tenth) and $PN = 6$ cm. Estimate the length NM from these data.

6.5. Brevet 5: The flat floor plan

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An architect provides the floor plan of a flat at a scale of $\frac{1}{50}$. The ceiling height is $2,50$ m.

- On the plan, the living room has a length of 12 cm. What is its actual length, in metres?
- On the plan, the bedroom has an area of 48 cm². What is its actual area, in m²?
- One piece of information in the problem was not used at all. Which one, and why is this normal?

6.6. Brevet 6: The 1/18 scale model car

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(Twin of exercise 4.2, for revision.) A scale model of a vintage car is at a scale of $\frac{1}{18}$.

- The actual car is $4,32$ m long. What is the length of the model, in cm?
- The actual windscreen has an area of $1,296$ m². What is the area of the model's windscreen, in cm²?
- The actual boot has a volume of $291,6$ L. What is the volume of the model's boot, in mL?

7. Going further (sorted question bank)

7.1. Geography: reading a map



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- a) On a map of France at a scale of $\frac{1}{2\,000\,000}$, the Paris–Lyon distance as the crow flies measures 19,6 cm. What is the actual distance, in km?
- b) By motorway, the journey is about 465 km. Why do the two answers differ? Which one does the map give?
- c) On a regional map at a scale of $\frac{1}{200\,000}$, a lake covers 4,5 cm². Calculate its actual area in km² (reinvent the powers of 10 from chapter 01: 1 km² = 10¹⁰ cm²).

7.2. Biology: under the microscope



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A photograph taken through a microscope is printed so that all lengths of the object are enlarged by a scale factor of $k = 400$.

- a) A cell measures 50 µm (1 µm = 0,001 mm). What is its size on the printed photograph, in cm?
- b) By what factor is the area of the image of the cell multiplied?
- c) Write 50 µm in metres, in standard form (scientific notation, chapter 01).

End of the exercise sheet: Detailed worked solutions in the document “Worked solutions to the exercise sheet”.