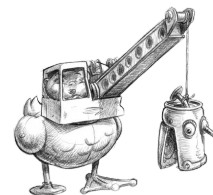


Test — Algebra: notable identities (Year 10)

Time allowed: 50 min

No calculator. For a proof, define precisely the chosen letter, then structure the reasoning with “Let... / Now... / Therefore...”. Every factorisation or expansion must be verifiable: the steps count. Marked out of 20.



Exercise 1: Nature, opposite and clever calculation

(4 marks)

- a) Is the expression $(7x - 12)^2$ a sum or a product? What about the expression $15x - 2(5x + 8)$? Justify in one sentence (the last operation performed). (1 mark)
- b) Give the opposite of the expression $13 - 5x$. (0,5 marks)
- c) Without writing out a multiplication, calculate 208^2 , stating the notable identity used. (1 mark)
- d) Without writing out a multiplication, calculate $171^2 - 71^2$ using a single multiplication, stating the notable identity used. (1 mark)
- e) Without writing out a multiplication, calculate 27×499 , stating the property used. (0,5 marks)

Exercise 2: Expand and simplify

(3,5 marks)

- a) $A = (4x - 15)^2$ (1 mark)
- b) $B = (6x + 11)(6x - 11)$ (1 mark)
- c) $C = 5x(6x + 7) - (7x - 1)(4x + 9)$ (1,5 marks)

Exercise 3: Factorise and check

(4 marks)

- a) Factorise by taking out the highest common factor: $D = 72x^2 + 56x$. (0,5 marks)
- b) Factorise: $E = 25x^2 - 196$. (1 mark)
- c) Factorise, then simplify the second factor: $F = (7x - 5)(6x + 1) - (7x - 5)(3x - 13)$. (1 mark)
- d) To factorise $G = 121x^2 + 198x + 81$, Sacha suggests the expression $11x(11x + 18)$. Show using a numerical counterexample that this factorisation is wrong, then factorise G correctly. (1,5 marks)

Exercise 4: Proof, two integers with a fixed difference

(4 marks)

We consider two integers that differ by 17, such as 1 and 18 (since $18 - 1 = 17$), and the difference of their squares, starting with the larger one: here $18^2 - 1^2$.

- a) Calculate $18^2 - 1^2$, then check that the result is a multiple of 17. Do the same with another pair of integers that differ by 17, of your choice. (1 mark)
- b) We would like to claim: “the difference of the squares of two integers that differ by 17 is always a multiple of 17”. Explain in one sentence why the examples from question a), even if numerous, are not enough to prove this. (0,5 marks)
- c) Prove this claim. Choose a letter to denote the smaller of the two integers, stating what it represents, then write out your proof using “Let... / Now... / Therefore...”. (2 marks)
- d) Is this difference always a multiple of 34? Justify. (0,5 marks)

Exercise 5: Layout of a square plaza**(4,5 marks)**

A square plaza $ABCD$ has side length $7x + 11$ metres, where x denotes a positive number. At its centre, a square pond $EFGH$ of side length 2 metres is installed. The rest of the plaza, shaded in the diagram opposite, is covered with paving slabs. Let $\mathcal{A}$ denote the area, in square metres, of this paved surface.

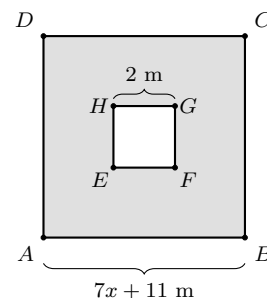


Figure not to scale.

- a) In this question only, take $x = 3$. Calculate the side length of the plaza $ABCD$, then calculate, without writing out a multiplication, the area $\mathcal{A}$ of the paved surface. (1 mark)
- b) Explain why, for every positive number x , we have $\mathcal{A} = (7x + 11)^2 - 4$, then expand and simplify $\mathcal{A}$. (1,5 marks)
- c) Factorise $\mathcal{A}$ by recognising a difference of two squares. (1 mark)
- d) In this question, take $x = 10$. Among the three expressions for $\mathcal{A}$ (original form, expanded form, factorised form), choose the one that gives the quickest calculation, justify this choice in one sentence, then calculate the area $\mathcal{A}$ without using a written method. (1 mark)

Bonus question**(+1 bonus mark)**

Find the positive integers k and p such that, for every number x , $4x^2 + 52x + 175 = (2x + k)^2 + p$. Deduce the smallest value that the expression $4x^2 + 52x + 175$ can take, and the value of x for which it is attained. Justify each step.

_____ END OF THE TEST _____