

Chapter 01B: Lesson guide

Algebraic expressions: notable identities, Year 10

Teacher document (detailed version of 2026-07-09), perennial (no reference to a specific year's timetable)

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This guide is designed to be *the only document open during the lesson*: each sheet contains the script, oral questions, problems used with their quick answers, the board plan and the word-for-word written summary. It complements the Work plan (the what) and builds on: the roadmap in the teacher version of the exercise sheet (core track highlighted: 1.3, 2.4, 3.3, 3.6, 4.1, 5.1, 5.4, 9.1), the audited V3 test, the duration audit (`audit_difficulte_parcours.md`), and the A/B formats and routines of the Year 10 teaching model.

Official safeguard for the chapter (quoted in the Starting point of the course notes): “no computational virtuosity is expected”. The difficulty of this chapter never comes from the length of the expressions: it comes from the traps (starting with the false identity $(a + b)^2 = a^2 + b^2$) and from the proofs. All the teaching conduct below stems from this: we never pad out a set of exercises to “keep pupils busy”; we stage an error or we build a proof.

0. Timeframe: independent of the timetable

This guide describes a linear sequence of standard 55-min slots. The statutory timetable for Year 10 is *3 hr 30 min per week*: the sequence must be adjusted to the actual timetable (slot duration, groups, bank holidays). Four slots are only a benchmark when the timetable provides them; lessons S0 to the final lesson run in order, without assuming a fixed weekly distribution.

Chapter budget: 12 slots, i.e. about 3 weeks. This is the first algebra chapter of the year: it prepares chapter 02 (zero-product equations), studied after the direct intercept theorem (Thales' theorem in France). *The test takes up a full slot* (S11: 50 min exam + buffer), counted within the 12; handing back papers takes the first 10 minutes of the next slot (opening of the next chapter, direct intercept theorem), not an extra slot. **Pace target:** by the evening of S8, the core track and the scheduled R selection have been worked on; Arthur is revisited at the opening of S9, then Brevet 6 (from the brevet, the French national exam at the end of Year 10) actually handed in opens S10. The H tasks remain a bank and never replace these reviews.

Three scheduling principles, valid for any timetable.

1. **If there is a double slot** (two consecutive lessons): pair an input lesson with its practice lesson (S1+S2, S3+S4, S6+S7). If there is none, nothing changes: the pairs also work one or two days apart.
2. **The test** (50 min) is placed in a single slot, preferably the *last of the week*: papers can be marked before the

next slot; handing back papers = the first 10 minutes of the following lesson (opening of the next chapter, direct intercept theorem).

- Never place a discovery lesson (S1, S3, S4, S8) right before a break (weekend, holidays) if consolidation cannot follow within 48 hr: the compared representations of S3 and the proof of S8 must be exploited while fresh.

Annual adjustment (the only thing to do at the start of each school year): map $S_0 \rightarrow S_{11}$ onto the actual slots following these three principles, and update the year-specific values ($2026^2 - 2025^2$ in 4.4 d). Everything else in the guide is stable.

0 bis. What this guide owes to mathematics education research

The principles retained inform the teaching conduct, but they do not prove that a timed script is optimal in this class. We therefore distinguish three statuses: **[S] established principle** (finding or description actually reported by research), **[T] reasoned transposition** (adaptation to Year 10 content) and **[D] local decision to be evaluated** (timing, frequency, threshold or organisation). Pink rows indicate a benchmark to be questioned; they do not constitute local experimental validation. In the table, the principle falls under [S]; the choice of conduct and its placement fall under [T], and any numerical calibration under [D].

Guide transposition or decision	Retained principle	Concrete trace in this chapter
Open practice lessons with a short drill, timed on a visible timer, without a calculator, then marked immediately; distinguish this fluency from a closed diagnostic test and from the initial construction of a concept	[S] A teaching tradition, described in a given cultural context, links speed, accuracy and practice. [T] Here, fluency is tested as reactivation; the timer is neither proof of understanding nor the sole objective. [D] We keep, lighten or untune according to observed accuracy and verbalisation. The first sorting task 1.1 includes 4 individual minutes but continues with a neriage; it is not reduced to a quick correction.	2.1 (identities, 6 min), 3.1 and 4.1 (clever calculation, 5 min) ; S0 distinguishes closed diagnostic test and inquiry 1.1
Build each input lesson around A SINGLE substantial situation (not a list of exercises); leave the useful pathway (data, trials, selected work, relationships and proof) written on the board until the summary	[S] The Japanese lesson unfolds in five stages: posing the problem, pupils' independent work, whole-class discussion, summary, then exercise or extension. It is this last stage, and not the summary, that is explicitly given as optional depending on available time. Experienced teachers make elaborate use of the board. [T] "Nothing is erased before the summary" is the local transposition retained here, not a research finding.	S1 (proving $k = a + b$), S3 (explaining then proving $(a + b)^2$) , S4 (the area proofs), S8 (three consecutive integers)
Run input lessons in four linked phases: opening question (hatsumon), independent work then circulating without correcting (kikan-shidō), ordered whole-class discussion (neriage), written summary never sacrificed (matome)	During independent work, the teacher circulates to understand and select: before the discussion, they choose the answers actually available and their order, so that comparison builds knowledge without fixing their authors in advance.	Sheets S1/S3/S4/S8; functional order of presentation in S3 and S8
Before each discovery lesson, anticipate the mathematical functions of several possible answers (more or less complete, possibly incorrect) and plan the relationships to make visible; during the lesson, select only from what actually exists	Preparation focuses on the contrasts needed for the mathematical goal, without an artificial quota of answers or errors. An essential missing pathway can be reconstructed collectively or proposed by the teacher under label T; it is never disguised as a pupil's work.	"Anticipation" section and R/T distinction on each sheet
When a real error is mathematically fruitful, make it inspectable without humiliating its author and have the class examine it; if the teacher wishes to submit a claim that did not appear, mark it T	Questioning remains focusing: same problem, requests for justification and comparison of reasoning. An error is possible material, not a mandatory step nor a belief attributed to the class in advance.	S2 (error or T), S4 (votes on 2.4), S7 (Louise's paper); script part 6
Work through families of exercises where ONLY ONE thing changes at a time (the given data, the direction, the dressing), and close each family with the routine verbalised by pupils: "what changes? what never changes?"	Variation theory (bianshi): one discerns the essential only if the non-essential varies against an invariant background. Three formats: one problem, several solutions; one problem statement, several variations; several problem statements, one method.	2.3 (one expanded form, several readings), 3.3 (12 traps, one method: the photographer), Brevets 3 and 6 (same expand/factorise/choose skeleton)

Guide transposition or decision	Retained principle	Concrete trace in this chapter
Insert a step between the worked example and the independent exercise: a fill-in-the-blanks example (“Over to you to complete”) filled in live, and each rule re-proved on the board by a pupil before any application	Progression by calibrated steps (pu dian): adjusting the distance between prior knowledge and new content, neither a leap nor over-guidance. The fill-in-the-blanks example is the intermediate step, absent from most handouts.	The “Over to you to complete” examples in the course notes $((3x - 2)(4x + 5), (3x - 8)^2, 81x^2 - 4)$; $k = a + b$ re-proved by a pupil in S2; area proofs redone in S3–S4
On any geometric result, require the figure (the dissection, the named areas) BEFORE writing the equality: “the drawing first, the calculation second”	Representing before calculating frees working memory and makes structure visible; a dissection proof can be read at a glance where the formula remains opaque. The working-memory cost of both pathways, model method and algebraic pathway, has been studied in detail.	Area proof of $(a + b)^2$ (S3), of the hollowed-out square $(a - b)^2$ and of the dissection $a^2 - b^2$ (S4); tablecloth (5.7) and garden (Brevet 7)
On true/false items and faulty papers, have the whole class vote by a show of hands BEFORE opening any justification; require a numerical counterexample for each “false”	Conceptions must be expressed to be addressed; voting engages the whole class before the explanation. One counterexample refutes; no number of examples proves: this is a learning objective in itself.	2.4 b, f, g in S4 (item-by-item vote, re-vote), 4.5 a, c in S5, 3.6 Louise in S7 (vote “is she right?”)
Install named and equipped metacognitive habits: here “the photographer” (how many terms? squares? the double product?) and reasoned checking: re-expand to establish the identity; substitute to look for a discrepancy, without ever taking a single match as a proof	Metacognition is taught through explicit labelling of actions; by repeatedly hearing the teacher’s questions, the pupil internalises them, and the teacher’s voice becomes their inner voice (the “inner teacher”).	The photographer (S6, displayed afterwards), then the refutation / proof distinction via 3.6 in S7; routines part 5
Insert problems where the raw result is NOT the answer, “the loss does not depend on the size”, a result equal to 0, or a factorisation whose status depends on available tools, and let the false lead play out before confronting it	Without explicit training, the pupil “suspends meaning”: they apply a procedure without asking whether the result answers the question. This is highlighted by comparing pairs of problems, one standard, the other realistic. Interpretation and the exact scope of a verdict are content in their own right.	4.5 (checking without calculating, S5), 7.2 (the plot of land: the loss is always 25 m ²), contrast $x^2 - 13 / x^2 + 81$ (S7), interpretation question in the test (Ex. 4 b)
Close each algebraic calculation with a check whose status is explicit: re-expand to establish a factorisation for any value; optionally substitute a simple value to detect a discrepancy, never to conclude from a single match	[S] In the textbooks studied, the looking-back step (verification) is unevenly made explicit. [T] The guide distinguishes general proof from numerical checking in models and pupil work. [D] The routine is maintained only if it improves error detection and explanation.	Re-expanding in the course notes examples; 3.6 (Louise’s paper) institutionalises the refute / prove distinction; the $x = 1$ substitutions in 2.2 and 3.2 are spot checks; in 3.3, the check is a re-expansion
Devote the first ten minutes of the lesson following any test to ONE anonymised real error, reformulated by the pupils and then generalised; code all papers and require a written, checked test correction	Jiangping (Shanghai): the start of the lesson exploits marked work (recap, presentation of the error, co-construction, generalisation). The personal error log (copying the error, the correction and the reason why) is here a local transposition of practices observed in certain Chinese contexts.	Sheet S11 (codes N/D/F/P/C); the first 10 minutes of the next chapter (direct intercept theorem); its opening fluency drills
Assign for homework only what has already been practised (never a new concept), and systematically exploit what comes back: as soon as it is handed out, each homework assignment states the lesson in which it will be marked and used	[S] Surveyed pupils attribute six functions to homework: improving understanding of concepts, revising and practising, improving problem-solving, preparing for tests, self-assessing and learning from mistakes, extending knowledge, and these six functions are the direct consequence of the pedagogical intention displayed by the teacher when setting the homework. [T] Hence the local rule: set on familiar ground, always exploited on return; a concept discovered alone at home and homework never marked cancel out the effect.	“Homework” column of each sheet; variants and explicit reviews of Brevet tasks 3, 5 and 6
Label each exercise by its function (M core track for all / R test level / H optional), run drills in class, and provide self-tests with worked solutions that the pupil uses alone before asking for help	Chinese textbook design: commented worked examples, graded practice and self-assessment; the self-test comes only after learning the content it draws upon.	M/R/H tracks of the exercise sheet; microtest part 3 after S4, multiple-choice quiz in S10; worked solutions to the exercise sheet handed out in S9, after revisiting Arthur, under a usage agreement

Guide transposition or decision	Retained principle	Concrete trace in this chapter
Before the test, have pupils experience real exam conditions (time limit, no calculator) and have the class judge pupils' solutions using the rubric question: "would this paper get full marks?"	Familiarity with item types reduces anxiety ("I won't panic"); performance assessment presupposes observation and judgement backed by explicit criteria, and self-assessment is among the alternative strategies actually trialled in Singapore.	S9 (Brevet 5 under exam conditions, with the marking grid), S10 (actual review of Brevet 6 + structured announcement of requirements)

0 ter. P1–P2 matrix and P9 audit of the chapter

The tables below are audit traces, not quotas: each row refers to an element actually present and makes it possible to observe the level reached.

Concept or method	Reactivated prerequisite	Organised variation (P1/P2)	Observable level
Nature of an expression	Order of operations, distributive property	Sum/product in visible or disguised forms; same value, two natures	Sorting 1.1 S0 → opposite/distributivity 1.2 → expand/factorise choice
Expanding double brackets	Distributive property, equality	Numerical case then letters; +/– signs; rule conjectured then proved	S1 proof → S2 proof reproduced → exercise sheet 1.3–1.5
Identities and factorisation	Square, area, common factor	Three identities in both directions, disguised forms, common factor before identity	Enlarged square S3 → guided example → 3.3/3.4 and 4.4

P9 dimension	Actual element in the chapter	Trace to observe
Concepts	Expression, term/factor, identity, equality and equivalence of two forms	Course notes parts 1–3; written summaries S0–S4
Techniques	Expand, simplify, factorise, recognise an identity, substitute to check	Exercise sheet parts 1–4; drills S0–S10, without jumping ahead
Processes	Two proofs of $(a + b)^2$, two pathways for Arthur, solving a created calculation program	S3; exercise sheet/worked solutions 5.1 and creation block
Metacognition	Question "what do I have / what do I want?"; re-expanding to establish an identity; test value to look for a discrepancy	S0, Louise's paper 3.6, test corrections
Attitudes	Not confusing examples with proof, verbalising a sign error, choosing a justified pathway	Arthur's program, true/false, grids S7–S10

1. Overview

#	Format	Title	Core of the lesson	Homework (set → exploited in)
S0	launch	01A feedback + nature	01A test feedback (0–10), inherited flash 01A 3.3 b)–d) , closed diagnostic, 1.1 explored then compared , enlarged square only handed over	R block: targeted 1.2 a)–c); M stops before 1.2 c); test repair → S2; square table → S3
S1	A	Expanding double brackets, proved	same Kadai for all: refutation → four products → proof via $k = a + b$; 1.3 a, b, d and exit ticket	R block: 1.3 c), e)–h) + 1.4 a); M does only 1.3 e)–h)
S2	B	Expanding practice	timed 1.3 e–h, S1 proof re-presented then varied, Kadai 1.4 C with complete answers compared , program 5.3	R block: 1.4 b), d); M does only b); Brevet 1 postponed until after S7
S3	A	From table to proof: $(a + b)^2$	prepared data → conjecture; search for dissections; area comparison; algebraic proof for all real numbers; 2.1 a, b, h	regular R block: 2.1 a), b), h) + 2.2 a), M without 2.2; light end of week 1 instead
S4	A	One same square, two remaining identities	inquiry on the shared square; readings of $(a - b)^2$ and $a^2 - b^2$; algebraic proof for all real numbers; 2.4 b, f, g voted on ; contrasted exit ticket	R block: rest of 2.1 + 2.2 c), e), g) + 2.4 a), c), d), e); M: rest of 2.1 only
S5	B	Clever calculation (the reward)	4.1, five fixed minutes , then Kadai: possible identity or genuinely economical tool? Compare $10,1 \times 9,9$ and 38×998 ; transfer 4.4 d; 4.5 a, c	R block: 4.2 a)–f), 4.3 a)–c), 4.5 b), d); M: 4.2 a), c), e) and 4.5 b), d)
S6	A	Factorising: the repeated block, then the photographer	family F_1 – F_3 ; valid factorisation or continued extraction; proof by re-expanding; opening of 2.3	R block: targeted 3.1 + 2.3 a) two expressions + 3.2 A + whole of 3.5; M without 3.2 A or the last two rows
S7	B	Four borderline cases + Louise’s paper	same sorting on 3.3 f, i, h, j: order, two routes, provisional limit, non-applicable identity; 3.6 Louise distinguishes refutation and proof	R block: 3.3 a)–e), g), k), l) + 3.4 a), b); M: a), c), g), l) + 3.4 a); light end of week 2 instead
S8	B	Checked cases → two general proofs	5.4 three consecutive integers : scope of the spreadsheet, choice of first integer or middle one, variation with four integers	R block: 5.1 a)–d) + 5.2; M stops after 5.1; 5.5 stays closed until the S9 Kadai
S9	B	Revisiting Arthur + compared proof + Brevet	comparison of actual pupil work on 5.1, then handout of worked solutions; 5.5 as a single problem; Brevet 5 under exam conditions + self-marking with the grid	Evening drills S9 + Brevet 6 → review in S10; 5.6 optional H
S10	B	Review, choice of form + bridge question	Brevet 6 actually reviewed ; three forms compared with no imposed winner; 4.1 at fixed duration; 9.1 $x^2 - 9 = 0$ with proof boundary; identity / equation; test announcement	Active memo + evening drills S10 + quiz + a single targeted review A/B/C; Brevet 7–9 optional H
S11	assessment	Test (50 min)	V3 template in force; codes N/D/F/P/C	test correction required (error log); end of week 3: exam-task weekend, Brevet 3

After each lesson, the short set on the sheet is set in the chosen homework slot, within the budget of section 2: “*Gammes du soir (evening drills)*” (5 questions rotating M1, 00, 01, 01A, Year 9 prior knowledge and the 01B skill already taught, answers provided upside down), see the homework policy in part 2. No full brevet task is doable before factorisation: the first weekend is therefore light (30 min of review/error log or rest). So is the second: Brevet 3 requires choosing the right form (V1 model, seen in S10) and the worked solutions to the exercise sheet to mark one’s work (opened in S9); the exam-task weekend therefore comes at the end of week 3. These variants replace regular homework; they do not add to it.

Pace target (teacher decision): by the evening of S8, the core track and the scheduled R selection have been encountered; Arthur returns for ten minutes in S9 because his homework done with the worked solutions closed is part of the learning process, not a bank to be rushed through. S9 runs Brevet 5 under exam conditions; S10 exploits Brevet 6 actually handed in, then builds the boundary towards chapter 02. Brevet 9 remains H: a task above test level does not replace the promised review of R work. If a lesson overruns, we cut from the drill or the bank, never

from the independent inquiry, the neriage, the matome or the announced review.

Official skills practised (benchmark for school reports and observations):

Skill	Where it plays out in the chapter
Explore	the proof $k = a + b$ (S1), the dissections of $(a + b)^2$ (S3), the spreadsheet conjecture (S8); individual inquiry before any discussion
Model	5.4 / 5.5 / Brevet 5 (translating “odd”, “consecutive” into algebra); 7.1 (energy), 7.2 (plot of land); 9.1 (equation)
Represent	area proofs: $(a + b)^2$, hollowed-out square, $a^2 - b^2$ dissection (S3–S4); tablecloth (5.7), garden (Brevet 7), mosaic (course notes)
Reason	2.4 true/false (counterexamples); 4.5 (invalidating without calculating); 5.1 / 5.4 (a finite list does not cover a claim about all integers); the photographer
Calculate	drills 1.1 / 2.1 / 3.1 / 4.1; 1.3, 2.2, 3.3; clever calculation (identities serving speed)
Communicate	standard wording “Let $n \dots$ Now $\dots$ Therefore” (S8); Brevet with the marking grid (S9–S10); verbalising invariants; justified votes

1 bis. Cross-reference exercise sheet ↔ lessons, and associated fast track

Each exercise run in class is placed in a specific lesson (its “Teacher” note in the teacher version of the exercise sheet gives the conduct; the lesson sheet gives the context); the right-hand column states where to send anyone who has finished immediately: always within the same theme, never “wait for the others”.

Exercise (conduct: Teacher exercise sheet)	Lesson · phase	For anyone who has finished (in order)
1.1 sum/product sorting	S0 · ph. 5–6	Justify an item not yet discussed, then invent a sum of products and a misleading square; do not read future formulas
1.3 expanding double brackets (a–d)	S1 · ph. 4	S1 block: c), then e)–h), then 1.4 a); M does only e)–h); 1.5 remains the extension
1.5 proof $k = a + b$	S1 · ph. 3 (prepared) → S2	2.7 “the formula for $(a + b + c)^2$ ” (H), only from S3 onwards: same block trick
5.3 the squared calculation program	S2 · ph. 4	explain the special case $(x + 1)^2 = x^2 + 2x + 1$, then 1.5 (block proof); do not open 2.6 (H) before S5
2.1 identity drills	S3 · ph. 5	in S3: a), b), h); S4 block: c), e), i), then d), f), g) on the Finish line; all nine M items are thus covered
2.2 with coefficients	evenings of S3–S4, R block	a) after S3; c), e), g) after S4; 2.3 waits for the factorisation lesson; 2.5 a)–b) remains optional H
2.4 voted true/false	S4 · ph. 5	b), f), g) voted on in class; a), c), d), e) in the evening R block: 7/7 coverage
4.1 timed clever calculation	S5 · ph. 1	4.2 a)–f) in the R block, then 8.1 (challenge: $n^2 - 9$ prime)
4.3 area between two squares	R block after S5	all of a)–c); M stops before 4.3
4.4 the champions (step-up run in class)	S5 · ph. 2–5	8.5 a) (cracking 24 999 999 without a calculator), then 8.5 b)
4.5 checking without calculating	S5 · ph. 6	7.1 braking energy ($(2v)^2$: the trap in context)
3.1 common factor	S6 · ph. 2	R block: 3.2 A and the six rows of 3.5; then 3.2 C; 3.3 waits for S7
3.3 four sorted cards	S7 · ph. 2–5	R block: 3.3 a)–e), g), k), l), then 3.4 a)–b); 3.7 a) remains optional H
3.6 Louise’s paper	S7 · ph. 6	invent a twin “false paper” and specify whether the proposed check refutes or proves
5.4 three consecutive integers	S8 · ph. 2–7	5.2 (programs A/B), then the creation block after 5.1; 5.5 stays closed until the S9 Kadai
5.1 Arthur’s program	homework after S8, then S9 · ph. 1	5.2 (programs A/B), then Brevet 8 (product of neighbours)
Brevet 6: three forms and justified choice	S10 · ph. 1–2	Brevet 9 (H), only after essential revision
9.1, bridge question checked / conjectured / proved	S10 · ph. 4–5	9.2 Ninon vs Grégory (H), without using an unestablished property

Permanent fast-track pathway (fast pupils or those ahead in the chapter), rules valid at any time:

1. **During an inquiry** (S1, S3, S4, S8): never “move on to the next exercise”; the instruction is always “find a SECOND proof” (algebraic vs geometric; expanding vs factorising) or “get ready to present at the board”. Being ahead is converted into depth.
2. **During a drill or a set of exercises**: the right-hand column of the table above, always an extension on the same theme (2.5, 2.7, 3.7, 8.5, and 4.6 “the square wins” for anyone who has finished clever calculation; 1.5 and 4.4 are now R as they are run in class, but can provide a second proof or the unrun item a)), never another chapter.
3. **From S5 until the test**: the “Going further” bank (part 8 of the exercise sheet) independently at the back of the classroom, in the order 8.1 → 8.2 → 8.3, then 8.4, 8.5 and 8.6 (the Pythagorean triple machine, echoing ch. 01A), under the agreement: without help, ungraded, getting stuck is expected. Milestones: 8.1 (an almost prime square) and 8.2 (the proof without words): offer ten minutes of board presentation to anyone who completes 8.3 (the conjugate expression, Year 11 prep).
4. **Chapter completely finished**: the ultimate step, **producing**: inventing a 13th trap for 3.3, a twin “false paper” of Louise’s, a “twin” Brevet task with its worked solution. A pupil who knows how to design the exercise has gone beyond the exercise.
5. **Peer tutoring**: possible but never imposed nor by default; a fast pupil is not a teaching assistant; if they help a classmate, they do so by asking the photographer’s three questions, not by giving the factorisation.

2. Equipment and logistics for the chapter

- **Photocopies**: course notes handout + exercise sheet handed out in S0 (one per pupil); the chapter’s “evening drills” sheet handed out in S0 with the handout (glued at the front of the exercise book, next to the Charter signed in ch. 00); **Memo** handed out in S10 only (“revision object” effect); test paper in S11; worked solutions to the exercise sheet kept closed until Arthur is revisited, then handed out in S9 under a usage agreement. This timing preserves an initial inquiry without a model while equipping independent work at the end of the chapter. **Usage agreement for the worked solutions**, stated when handed out and non-negotiable: 1) try and write out your work first, open the worked solutions afterwards, never the other way round; 2) compare, identify your error, and if it is a conceptual error it goes into the error log; 3) the “model answer” boxes are used to judge your own writing (“would my paper get full marks?”); 4) the evening Brevet is always written with the worked solutions closed, then checked afterwards. Rationale: Chinese textbooks provide answers to self-tests precisely so that the pupil looks for their own mistake alone before asking. Warning sign: homework suddenly free of crossings-out.
- **Organisation of pupil materials**: two books + one log. Notebook: the written summary of each lesson, dated, copied from the board in the order of the reasoning pathway; the Japanese board (bansho) is designed to be copied: the written summary tells how we came to know, not just what to know; the handouts are glued into it. Exercise book: evening drills, exercise sheet, evening Brevets; for the evening Brevets, no rough paper: pupils write directly as in the brevet exam. Error log (local transposition of observed practices of reviewing homework; to be evaluated on the quality of corrections): after each test and each failed microtest, the pupil records the problem in one line, THEIR error, the correction, and “I made a mistake because...”; it is filled in from the first test onwards and re-read before each subsequent one. The two errors that fill this log in this chapter: $(a + b)^2 = a^2 + b^2$ and confusing $a^2 - b^2$ with $(a - b)^2$.
- **The board (bansho): standard zoning for the year**, announced from S1 and identical in every input lesson, LEFT: the problem and data (and the rough work zone, the only erasable zone); CENTRE: pupils’ work, displayed in the order chosen while circulating, including incorrect ones (crossed out, never rubbed out); RIGHT: comparison and then institutionalisation (the written summary). Three rules of use: 1) nothing is erased before the summary, the board is the collective memory of the reasoning; at the end of the lesson, a pupil must be able to read it and reconstruct the pathway; 2) deferred copying, during independent work and whole-class discussion pupils do not copy, they think; they copy only the summary, at the end; 3) **representations are only valuable when linked**: an area figure and a formula side by side only help if the link is explicitly drawn (“which piece of the drawing is the $2ab$?”). The board plan for each input lesson is prepared before the lesson (bansho keikaku). The twelve ideal boards in this guide also exist in large format in the separate document TableauxProjection (landscape, one board per page), to be projected as a summary check or backup; the live chalk board remains the norm: it is its construction in front of the pupils that carries the reasoning, not its final state.
- **Mini-whiteboards** (or reused half-sheets): essential for the votes in S4 and the exit flashes in S1, S3, S4, S7. In S3, they collect the final expansion without turning the construction of the proof into an opinion poll.
- **Visible timer** for the individual time on sorting task 1.1 (4 min, followed by a comparison and not a simple correction), then for drills 2.1 (6 min), 3.1 (5 min), **4.1 clever calculation (5 fixed min)** and the 10 min of Brevet 5 under exam conditions in S9. On clever calculation, record at fixed duration the number of pupils with at least 6 correct answers out of 8, then 8 out of 8; use this exact indicator again in S10. We measure progress in accuracy under a stable constraint, not a vague “class time”.
- **“The photographer” poster** (A3 or drawn on the board) from S6 onwards: how many terms? are the outer terms

squares? is the middle term the double product?: it stays on display until the test.

- **Computer room or projected spreadsheet** for S8 (5.4: conjecturing divisibility by 3); failing that, a spreadsheet on the projector is enough.
- **Teacher's logbook:** note in each lesson the 2–3 errors actually observed (they feed into the next fluency drill and test feedback).
- **Seating plan for circulating (kikan-shido, kit K3):** print the classroom seating plan with the anticipated P functions of the peak sheet; while circulating, note the answers actually observed R1–Rk, their degree of completion and possible links, then decide the order of the neriage. The categories designate neither pupil levels nor mandatory errors. First set up in S0, reused at peaks S1, S3, S4, S5, S8.
- **Observable homework budget [D]:** local decision to be evaluated over four weeks. The three tiers remain Learn (active re-reading), Evening drills (self-marked reactivation) and Do (already practised), but they are governed by a cap, not a uniform quota. Track *M aims for 35–40 min* and may stop at the threshold while leaving a trace of inquiry, never penalised; track *R aims for 40–50 min*; track *H is optional* and creates no workload debt. *No evening exceeds 60 min*: nothing spills over into the next evening and no exercise is added to “catch up”. The first two weekends are light: no full Brevet task can precede factorisation, and Brevet 3 waits for the choice of form (V1 model, S10) and the opened worked solutions (S9); the exam-task weekend therefore comes at the end of week 3. In both cases, the announced variant replaces regular homework and never creates a cumulative second load. For four weeks, the logbook records actual duration (M/R), stops at the threshold and two pupil feedbacks; the calibration is then kept, lightened or shifted. The value of review and error feedback is established; this numerical calibration remains a local decision to be tested.
- **Weekly flash and 01A → 01B stitching:** at the start of S0, the 5-min formative flash reuses the exact structure of **3.3 b)–d)** from chapter 01A (reverse direction: finding *k*), with possibly changed data; it replaces any other fluency drill and is ungraded unless announced beforehand with criteria and an opportunity to retake. The actual R/A/K/D/U codes from the 01A test, known only after marking, then steer the first three lessons: in S0, the anonymised error is chosen from a dominant code; in S1 and then S2, a short question from the still-fragile dominant code **replaces** an item in the planned fluency drill, without adding to it. The teacher records in the logbook the code revisited and the observed result. After S4, possible flashes are chosen from the items of 2.2 actually worked on, then **3.4 a** after S7. Sheet S11 in turn designates exercise **5.4** as the legacy passed on to the next chapter.
- **Rotating collection:** carried over from ch. 00: *3–5 exercise books observed in each lesson*, in a displayed order (each pupil is checked roughly every two weeks); we look at the trace of inquiry (evening drills ticked, trials, crossings-out, Brevets written with worked solutions closed), never neatness, “work with crossings-out is good work”.

3. The S0 diagnostic, item by item

The 10 fluency skills in the course notes (p. 2) serve as a silent X-ray. This chapter relies on a narrow, precise set of prerequisites: distributive property, simplifying, opposite of an expression, common factor (learnt in Year 9). Reading the errors:

Failed item	What this indicates	Immediate remediation
1) $3(x - 4)$ (ans. $3x - 12$)	distributive property not secure → core prerequisite for the chapter	call on these pupils first in S1; refer to 1.2 a)
2) simplify $6x + 5 - 4x + 2$ ($2x + 7$)	like terms not collected	1.2 c) in class; send them to the board
3) opposite of $7 - 2x$ ($-7 + 2x$)	the $-$ sign only changes one term (trap $-7 - 2x$)	1.2 b) as a priority; this is the test error in Ex. 1 b
4) $2^3 \times 2^4$ ($2^7 = 128$)	review of ch. 01 (rules for powers)	note the names; outside the 01B core, but useful in 5.6
5) $\frac{42}{56}$ ($\frac{3}{4}$)	review of ch. 01 (lowest terms)	quick warm-up at the start; not critical here
6) 0,0035 in SF ($3,5 \times 10^{-3}$)	review of ch. 01 (standard form)	same; retention check
7) $(-5)^2$ vs -5^2 (25 ; -25)	role of brackets / of the sign	drills will bring it back; foreshadows the trap $(2x)^2$ vs $2x^2$
8) factorise $4x + 28$ ($4(x + 7)$)	no common-factor reflex → prerequisite for S6	3.1 in class targets them; identify them right away
9) $x^2 - 3x$ for $x = -2$ ($4 + 6 = 10$)	substitution + signs: checking tool for the chapter	will be reused at every “I check with $x = 1$ ”; note the names
10) similar triangles: angles (equal)	review of 01A	no direct link; simple memory maintenance

Alert threshold: a pupil who fails items 1, 2, 3, 8 (the **Year 9 algebra block**: expand, simplify, take the opposite, factorise) belongs on the full M track + close monitoring (see section 9). Write down the list from S0: these are the pupils to support closely on the line of four signed products in S1 and on the common factor in S6.

4. Detailed lesson guides

S0: Launch, diagnostic and the nature of an expression (55 min)

Pupil objective. Reactivate Year 9 prior knowledge; identify the main operation in an expression; distinguish its structure from its value; understand the M/R/H tracks. **Skills:** calculate, investigate, communicate.

Equipment. Marked 01A test paper; course notes folded over the answer strip; graded exercise sheet; “Gammes du soir (evening drills)” sheet; half a sheet of paper per pupil for the diagnostic; notebook; timer; S0 bansho in front of you.

Managing the transition. Returning the test and the carried-over flash activity 3.3 b)–d) from chapter 01A fit within the 55 minutes: the flash activity is formative, lasts 5 minutes and replaces any other fluency drill. Prepare before pupils enter an anonymised real error, the R/A/K/D/U codes, the flash activity and the half-sheets. Handouts are placed face down or folded; no diagnostic answer may be visible before collection.

Phase 1 (0–10): Returning test 01A, genuinely integrated

Hand back the papers; project ONE anonymised real error. Two pupils rephrase it, the class locates the first invalid line, repairs that line and then formulates a transferable rule. Each pupil writes in their error log the R/A/K/D/U code and ONE priority correction. At 10 minutes, photograph or record the “error (repair) rule” triplet, then erase the board: the two chapters must not mix on the board. The personal correction is due for S2, not the next day.

Phase 2 (10–15). Carried-over 01A flash: 3.3 b)–d), in reverse

Project three questions with the same structure as 3.3 b)–d) from chapter 01A, using the original data or modified data: find the scale factor k from a ratio of areas, or move from one area to the other using k^2 . Five minutes, formative by default, immediate correction. Have pupils first write “effect on areas: $\times k^2$ ”, then conclude; do not add any other fluency drill.

Phase 3 (15–17): Targeted working agreement, without a guided tour of the course notes

Have pupils open only the **Starting point**. Read the sentence “no computational virtuosity is expected”, then have them point to the icons on the exercise sheet.

Q1. “What is the difference between M, R and H?”, Expected: M = shared priority core; R = practice matching the test expectations, selected and non-cumulative; H = optional extension. Say this in three sentences, without going through the whole set of course notes.

Q2. “What will make this chapter demanding if it is not length?”, Expected: recognising a structure, justifying a transformation and checking an equality. The formulas are not read out: they will be built during the scheduled lessons.

Phase 4 (17–27). Fluency skills: a genuine closed diagnostic

The course notes stay folded over the answer strip. The 10 questions are answered individually on a half-sheet for 7 minutes, without a calculator. Collect all half-sheets BEFORE having pupils unfold the answer strip; then allow 3 minutes of silent self-checking. No item is taught during this time: after the lesson, the teacher codes successes and errors, especially the block **1, 2, 3, 8**. An unattempted answer is thus distinguished from a copied answer.

Quick answers (teacher): 1) $3x - 12$ 2) $2x + 7$ 3) $-7 + 2x$ 4) $2^7 = 128$ 5) $\frac{3}{4}$ 6) $3,5 \times 10^{-3}$ 7) 25 and -25 8) $4(x + 7)$ 9) $4 + 6 = 10$ 10) equal in pairs.

Phase 5 (27–38). 1.1: individual work then collecting answers

Write the **Kadai**: “Without expanding, how can we decide whether an expression is a sum or a product?” Four minutes working alone on the ten expressions, circling the operation that applies to the whole expression; three minutes in pairs to compare justifications; four minutes to collect answers. Walk around with the seating plan, spot specific roles, then record actual answers on the board under R1, R2, ..., without names or initials.

Quick answers (teacher): a) sum b) product c) product d) sum e) sum f) product $((2x - 3) \times (2x - 3))$ g) sum h) sum i) product j) product.

Point of rigour. Here, “last calculation” means the **main operation**: the one that applies to the whole expression after reading brackets and order of operations. An isolated square is treated as a product because $E^2 = E \times E$;

without this explicit convention, saying “the last calculation is a square, therefore a product” would be illogical.

Phase 6 (38–46). Neriage: four roles, not four corrections

Order the answers by role:

P1✗ “ $(2x - 3)^2$ is a sum because there is a minus sign”: keep the argument, then ask whether this sign applies to the whole expression or only inside the square;

P2✓ rewriting $(2x - 3)^2 = (2x - 3) \times (2x - 3)$: this makes the product visible;

P3✗ “ $x(x + 1) + 3(x + 2)$ is a product since it contains multiplications”: have pupils circle the outer addition;

P4✓ Explicitly write $5(x + 4) = 5x + 20$ for all x on the board. Ask: “does the distributive property change the values obtained? does it change the written structure?” Expected: no for the values, yes for the main operation; the first expression is a product, the second a sum. This simple distributive property, already learnt, replaces expanding $(2x - 3)^2$ and does not give away any notable identity.

Q3. “Why does this sorting determine which action to choose?”, Expected: expanding turns a product into a sum; factorising turns a sum into a product, without changing the values obtained.

If a role is missing. Ask a prompt question to the whole class and record the suggestion under the letter T. Never introduce a “prepared script” presented as pupil work.

Phase 7 (46–49): Handing over the square problem, answer deferred until S3

Read only the situation of the square vegetable patch of side 10 m increasing to 12 m and Karim’s claim: “the area gains $2^2 = 4 \text{ m}^2$ ”. Have pupils write the expression comparing the two areas, without carrying out the calculation and without saying whether Karim is right. The numerical table and the conjecture are to be prepared for S3, not for S1.

Quick answers (teacher): Expected results reserved for S3, not to be displayed: $a=10, b=2 : 144/104$, gap 40; $a=3, b=4 : 49/25$, gap 24; $a=5, b=1 : 36/26$, gap 10; $a=7, b=10 : 289/149$, gap 140. Each gap equals $2ab$.

Phase 8 (49–55): Protected matome and copying

First have the pupils formulate the rule, refine it, and then compare it with the written summary. Return to $x(x + 1) + 3(x + 2)$: despite the internal products, the addition applies to the whole expression; it is a sum. The vegetable patch question remains visible and unanswered.

IDEAL BOARD (final state): to project or reproduce

LEFT: the problem and the working agreement

Kadai: “Without expanding, how can we decide whether an expression is a sum or a product?”

Three tracks: M = core for all; R = test level; H = going further.

Diagnostic: answers handed in before opening the answer strip.

Question kept open for S3: a square of side 10 m increases to 12 m; is the area gained equal to 2^2 ?

CENTRE: anonymous answers to sorting 1.1

1.1: $3x + 2$ sum; $5(x + 4)$ product; $x^2 - 9$ sum; $(2x - 3)^2$ product ($= (2x - 3)(2x - 3)$)

“ $(2x - 3)^2$ is a ~~sum~~ because of the $-$ ”
 ~~$(2x - 3)^2 = (2x - 3) \times (2x - 3)$: the square is read as a product.~~

“ $x(x + 1) + 3(x + 2)$ is a product”: the addition applies to the whole expression.

T: $5(x + 4) = 5x + 20$ for all x ; on the left a product, on the right a sum.

RIGHT: key takeaways

Main operation: the one that applies to the whole expression, after brackets and order of operations.

$+/- \rightarrow$ sum (of terms)

$\times \rightarrow$ product (of factors)

$E^2 = E \times E$: a square is read as a product.

We classify an expression, not its value.

PRODUCT $\xrightarrow{\text{expand}}$ SUM
 SUM $\xrightarrow{\text{factorise}}$ PRODUCT

Written summary.

📎 TO COPY INTO THE NOTEBOOK (word for word)

The main operation is the one that applies to the whole expression, once brackets and order of operations have been respected. If it is $+$ or $-$, the expression is a sum of terms; if it is $\times$, it is a product of factors. A square is read as a product: $E^2 = E \times E$.

We classify an expression, not its value: $5(x + 4)$ is a product and $5x + 20$ is a sum, even though these two expressions have the same value for all x . **Expanding** turns a product into a sum; factorising turns a sum into a product.

Homework, capped workload. The family box shows the full R prescription. M track: learn the written summary and the six words term, factor, sum, product, expand, factorise; do the five S0 evening drills; finish 1.1 if needed; in 1.2, do a) and the first two opposites of b), then stop before c). H track, optional: invent three expressions including a sum of products and a misleading square. The priority correction from test 01A is due for S2. The “growing square” table and a conjecture are to be prepared for S3: they are not due for S1.

Learn by heart: the written summary and the words term, factor, sum, product, expand, factorise.
Evening drills: “Evening of S0” block, upside-down answers to be checked after trying the questions.
Do: 1.2 a), the first two opposites of 1.2 b), then 1.2 c).
Do: complete the growing square table, course notes p. 2, and write a conjecture for S3.
Rework: one coded error from test 01A in the error log for S2.
Finish: 1.1 if needed.
Optional: invent three expressions to sort, including a sum of products and a misleading square.

If running late. Postpone only the handover of the square problem; announce it directly for S3 instead. Individual sorting time, neriage and matome cannot be cut.

Success criteria... each pupil can distinguish M/R/H, identifies the main operation in f) and h), explains why E^2 is read as a product and distinguishes “same value” from “same structure”. After reading the half-sheets, the teacher draws up the support list for items 1, 2, 3 and 8; it cannot be considered reliable before this review.

Teaching note: The lesson separates assessment from teaching: the diagnostic is closed and then analysed afterwards, and comparison is based on actual pupil answers. The bansho preserves error, refutation and relationship without attributing fictitious answers. Comparing $5(x + 4) = 5x + 20$ uses only the already learnt distributive property and preserves the unknown for S3. The square problem is handed over and then deferred: the prepared data become the shared class material for the conjecture, representations and proof.

Management kit: initial set-up. From phase 5 onwards, tick off the roles on the seating plan to choose the speaking order. During the summary phase, ask “which operation applies to the whole expression?”, have pupils refine the sentence, and then compare it with the written summary. Record R1–Rk without identifiers; any contribution constructed by the teacher is marked T.

Optional introductory variant (outside time budget): letting the letter emerge as an abbreviation

The letter in this chapter can be invented by the class rather than handed down. The planned S0–S1 sequence remains the default route; here are two short hooks to slip into S0 or S1 when time permits.

A. The letter as an abbreviation (5 steps). (1) “Choose a number, add it to its double: you get its triple. True for 5? for 12? for 3,7?” (2) Challenge: “write this property for any number, in fewer than ten symbols, without the word ‘number’” (3) Collect pupil ideas, drawings ($\square + 2\square = 3\square$), shortened words, initials: all are legitimate. (4) Reveal: mathematicians chose a letter, and $x + 2x = 3x$ compresses infinitely many sentences into one. (5) Connect to the identity “=” sign (guided reading of the course notes).

B. Dots and matchsticks (“guess my method”). Dots arranged in a square, 7 per side (24 in total): “find how many there are without counting one by one; write your method as an expression”; then “same figure with 10 per side, reuse your expression”. Heart of the neriage: guess a classmate’s counting method from their expression alone (6×4 ; $7 \times 2 + 5 \times 2$; $7 \times 4 - 4$; target error $7 \times 4 = 28$, corners counted twice). Matchstick variant (8 squares in a row $\rightarrow$ 25 matchsticks: $4 + 3 \times 7$, $4 \times 8 - 7 \dots$), then 30 squares: drawing becomes impractical, the expression becomes necessary. Close with the ritual question: “what changes? what never changes?”.

S1: Expanding double brackets, proved (format A, 55 min)

Pupil objective. Establish, for all real numbers a, b, c, d , the formula for expanding double brackets from the distributive property, then apply it without losing the signs. **Skills:** investigate, reason, calculate.

Equipment. Course notes part 2; exercise sheet 1.3; mini-whiteboards.

Phase 1 (0–8): Homework fluency and review of a 01A code

First project a short question matching the dominant R/A/K/D/U code that is still fragile; it replaces one of the three expressions in 1.2 a), without lengthening the phase. Then go over the remaining two expressions of 1.2 a), followed by the first two opposites of 1.2 b).

Quick answers (teacher): $3(2x + 5) = 6x + 15$; $-4x(6 - 3x) = -24x + 12x^2$; $7(3 - x) + 2x = 21 - 5x$; opposite of $5 - 4x$: $-5 + 4x$; opposite of $3x - 7$: $-3x + 7$.

Briefly revisit the opposite trap (item 3 of the diagnostic); the in-depth logical comparison of true or false statements is reserved for S4:

Q1. “Opposite of $5 - 4x$? What exactly does the number -1 multiply?” Expected: $-(5 - 4x) = -1(5 - 4x) = -5 + 4x$; the factor -1 multiplies each of the two terms.

Phase 2 (8–19): Kadai and investigation (hatsumon then kikan-shidō)

Write the Kadai: “How many products must be written to expand $(a + b)(c + d)$, and how can we justify this using only the distributive property?” Individual work 4 min, then pairs 4 min. While circulating, code the answers without names, distinguish result, rule of thumb and proof, and prepare the speaking order. Do not give either the four arrows or the letter k . For fast finishers: “can you write a chain of equalities where each step is justified by a property you already know?”

Phase 3 (19–34). Neriage: refute, distinguish procedure from proof, then prove

The formula is new, but it is not an additional principle: it follows from the distributive property. This is the first proof constructed by the class in this sequence; have pupils name the properties used.

Anticipated answers (speaking order):

P1X “ $(a + b)(c + d) = ac + bd$ ”. If nobody suggests it, the teacher may put it forward as statement T to be tested, without presenting it as pupil work;

P2✓ four arrows and $ac + ad + bc + bd$, correct result but with no written reason;

P3✓ proof using $k = a + b$; if the idea of naming the bracketed group does not emerge, the prompt comes from T and the subsequent steps remain credited to actual pupil answers;

P4✓ possible symmetrical proof using $h = c + d$, contingent and never fabricated.

Q2. “For $a = b = c = d = 1$, do both sides of the proposed equality have the same value? What does this test prove, and what does it not prove?” Expected: the left-hand side equals 4 and the right-hand side 2; this counterexample is enough to refute the equality claimed for all real numbers, but it does not yet give the right formula. Conversely, a value that works would prove nothing in general.

Q3. “If we let $k = a + b$, which property justifies each equality? Why can we then swap bc and ad ?” Expected: $k(c + d) = kc + kd$, then $(a + b)c = ac + bc$ and $(a + b)d = ad + bd$; three applications of the distributive property. Finally, $ac + bc + ad + bd = ac + ad + bc + bd$ by commutativity of addition.

Naming the law (handing over notation). Once the proof is complete, the class may suggest a name for one minute; write it next to the official name “expanding double brackets (double distributive property)” for this lesson only. The name never replaces the justification.

IDEAL BOARD (final state): to project or reproduce

LEFT: problem

Kadai: How many products must be written to expand $(a+b)(c+d)$, and how can we justify this using only the distributive property?

For real numbers a, b, c, d .

Do a rule of thumb or four arrows already provide a proof?

CENTRE: pupil work

R1 or T: $(a+b)(c+d) \neq ac+bd$
 $a=b=c=d=1$; $4 \neq 2$; the general equality is refuted.

R2: four arrows: $ac+ad+bc+bd$.

R3 or T, proof: $k=a+b$;
 $(a+b)(c+d)=k(c+d)=kc+kd$
 $= (a+b)c + (a+b)d = ac+bc+ad+bd$
 $= ac+ad+bc+bd$ (commutativity of addition)

RIGHT: summary (written summary)

For all real numbers a, b, c, d :
 $(a+b)(c+d) = ac+ad+bc+bd$.

Method: four products with the sign of their two factors, then simplify.

Ex. $(x-3)(2x+5)$
 $= 2x^2 + 5x - 6x - 15 = 2x^2 - x - 15$.
 For $x=1$, both expressions equal -14 :
 spot check, not a proof.

Phase 4 (34–47). Signed variation: 1.3 a), b), d) for all

Have pupils work on a), b) and d); c) is reserved for fast finishers. For each of these products of two brackets, require the line of four products with the sign of their two factors before any simplification. After d), have pupils verbalise: “the term is -3 , not 3 ; its sign goes into each of the two products”. Items d), e), f) are the official examples: they will return in warm-ups, but e) and f) use the distributive property, not four arrows.

Quick answers (teacher): a) $(x+3)(x+5) = x^2 + 8x + 15$ b) $(x-3)(2x+5) = 2x^2 - x - 15$ c) $(3x+1)(2x-7) = 6x^2 - 19x - 7$
 d) $(2x-3)(5x+7) = 10x^2 - x - 21$.

Q4. “Does checking b) or d) with $x=1$ prove the expansion?”, Expected: no; equality for a single value is a spot check. The general proof relies on the distributive property. On the other hand, a discrepancy for one value would be enough to refute the claimed expansion.

Phase 5 (47–53): Protected matome, formulated then copied

From P2 and P3, have pupils state the domain, the formula and its justification, then display the written summary. The proof using $k=a+b$ remains visible during copying.

TO COPY INTO THE NOTEBOOK (word for word)

For all real numbers a, b, c and d :

$$(a+b)(c+d) = ac+ad+bc+bd.$$

Each term in the first factor multiplies each term in the second: four products, written with the sign of their two factors, and then we simplify. This formula is not an extra rule: it is proved by letting $k=a+b$, applying the distributive property three times, and finally swapping two terms of the sum (addition is commutative).

Example. $(x-3)(2x+5) = 2x^2 + 5x - 6x - 15 = 2x^2 - x - 15$.

A numerical test can refute an equality claimed for all numbers; it never proves it.

Phase 6 (53–55): Exit check

Mini-whiteboard: expand $(x-4)(x+1)$ writing the line of four products. Expected: $x^2 + x - 4x - 4 = x^2 - 3x - 4$.
 Observe the intermediate line and the final answer separately; note who still needs the arrows.

Differentiation. Fragile pupils (items 1–2 of the diagnostic): arrows required only on products of two brackets, signs written inside the terms. Fast finishers: do 1.3 c), then look for the alternative proof by letting $h=c+d$.
 For S2, a volunteer rephrases the S1 proof in their own words; they do not discover it alone at home.

Homework, capped workload. The family box shows the full R prescription: 1.3 c), e)–h), then 1.4 a). M track: reread the proof and the example; learn the formula and the sentence “four products with their signs, then simplify”; do the five S1 evening drills; do only 1.3 e)–h), then stop before 1.4 a). On e) and f), use the distributive property; on g) and on the product inside h), write the four products. H track, optional: prepare an oral explanation of 1.5 or the proof using $h=c+d$. The part 2 micro-test remains an available self-test, not an extra task. Announce the 1.3 fluency drill and the targeted 01A review for S2.

Read: course notes pp. 8 to 9, proof of expanding double brackets and model D1.

Learn by heart: $(a + b)(c + d) = ac + ad + bc + bd$; four products with their signs, then simplify.

Evening drills: “Evening of S1” block, upside-down answers to be checked after trying the questions.

Do: 1.3 c) and e) to h), then 1.4 a).

Optional: prepare an oral explanation of 1.5 or redo the proof by letting $h = c + d$.

The next lesson will begin with a fluency drill on 1.3 and a targeted review of test 01A.

If running late. Drop P4 first, then item d) of the practice. Investigation, P1–P3 comparison, general proof, six minutes of matome and exit check cannot be cut. A missing approach is never replaced by a fake pupil script; any input from the teacher is marked T.

Success criteria... on the exit check, over 80 % obtain $x^2 - 3x - 4$ with the line of four products; everyone can explain that the general proof comes from the distributive property and not from the $x = 1$ test.

Teaching note: A single problem carries the error, the procedure, the proof and the variation, so that pupil productions remain simultaneously comparable. The board keeps the refuted equality, the justifications and the matome until photographed. The sequence moves from the conjectured result to the proof, then to the signed variation and independent practice, without confusing example and proof.

S2. Expanding practice: speed, mixed expressions, and a first calculation program (format B, 55 min)

Pupil objective. Correctly subtract an expanded product; compare a complete error with a safe method and, if it arises, with another valid route; distinguish algebraic transformation, refutation and spot check; then translate a calculation program into an expression in x . **Skills:** calculate, reason, communicate.

Equipment. Exercise sheet 1.3, 1.4 C and 5.3; mini-whiteboards; timer.

Phase 1 (0–8): 1.3 fluency and review of a 01A code

A short question matching the second R/A/K/D/U code that is still fragile replaces item e), without lengthening the phase. Then do f), g) and h), without a calculator, and correct selectively: state the answers, but write out only the lines that require justification. Item f) practises the opposite of a bracketed group; g) a product of two brackets; h) an expansion followed by cancellation.

Quick answers (teacher): e) $-4x(6 - 3x) - 8x^2 = 12x^2 - 24x - 8x^2 = 4x^2 - 24x$ f) $3(2x + 1) - (6 - x) = 7x - 3$ g) $(-3x + 5)(2x - 3) = -6x^2 + 19x - 15$ h) $(x + 6)(x - 2) - x^2 = 4x - 12$.

Q1. “In f), what exactly does the factor -1 multiply? In h), which terms cancel out only after expanding?”, Expected: $-(6 - x) = -6 + x$; in h), the two x^2 terms cancel out after the product has been expanded.

Phase 2 (8–17): Checked recap of the S1 proof, then variation

A volunteer presents again the proof built in S1; the class checks each equality and names the three applications of the distributive property followed by commutativity. If there is no volunteer, have the class reconstruct the chain collectively from its starting point $k = a + b$ alone: do not attribute the proof to the teacher or to a fictitious pupil. The proof stays in the CENTRE. Then reconstruct:

$$k(c - d) = kc - kd = (a + b)c - (a + b)d = ac + bc - (ad + bd) = ac - ad + bc - bd.$$

Q2. “Where must the subtracted group remain visible? Which property justifies each equality?”, Expected: the method is identical; subtraction requires keeping $(ad + bd)$ grouped before distributing -1 . The final order is obtained by commutativity of addition.

Phase 3 (17–33). Kadai 1.4 C: investigation and neriage

Write: “How can we expand $C = 3x(x - 3) - (x + 7)(4x - 2)$ without losing the sign in front of the second product, and then check the result?” Individual work 4 min, pairs 3 min, then discussion 9 min. While circulating, select complete solutions rather than successive lines from a single script.

Anticipated roles:

P1. complete incorrect answer $-x^2 + 17x - 14$, obtained by changing only the sign of $4x^2$; if it does not appear, T puts this equality forward for testing without attributing it to a pupil;

P2. complete safe method: expand the second product inside brackets, then distribute -1 , and simplify to $-x^2 - 35x + 14$;

P3. possible alternative method: absorb the minus sign into the first factor, $-(x + 7) = -x - 7$, then expand;

P4. comparative check at $x = 1$: initial expression $= -22$, P1 $= 2$, P2 $= -22$.

Q3. “What does the discrepancy $2 \neq -22$ refute? What does the agreement $-22 = -22$ prove, or fail to prove?”, Expected: the discrepancy refutes P1 as an identity; the agreement does not prove P2, whose proof is the chain of justified equalities.

Q4. “In method P3, where did the factor -1 go?”, Expected: $-(x + 7) = (-1)(x + 7) = -x - 7$; P3 is correct but contingent. Do not fabricate it if neither pupil work nor a whole-class prompt brings it out.

Phase 4 (33–44). Calculation program 5.3: from the particular to the general

Read the program (choose x ; add 1; square the result; subtract the square of the starting number). Do a) and b) quickly, then translate into algebra before calculating:

$$P(x) = (x + 1)^2 - x^2 = (x + 1)(x + 1) - x^2 = x^2 + x + x + 1 - x^2 = 2x + 1.$$

This line is a special case obtained by expanding double brackets; it will serve as data in S3 to generalise the square of $a + b$. Factorising via $a^2 - b^2$ is neither requested nor shown here: it will be the second review method in S8. Solve $2x + 1 = 15$ only if time permits.

Quick answers (teacher): a) $x = 1 : 2^2 - 1^2 = 3$ b) $x = 2 : 3^2 - 2^2 = 5$ c) $P(x) = 2x + 1$ d) $2x + 1 = 15 \Rightarrow x = 7$.

Q5. “Do the two trials 3 and 5 prove the formula $2x + 1$? Which line holds for all x ?”, Expected: the trials suggest the pattern; the chain of algebraic equalities establishes the formula for all real numbers. If x is an integer, $2x + 1$ is odd; for an arbitrary real number, parity does not apply.

IDEAL BOARD (final state): to project or reproduce

LEFT: Kadai and compared answers

Kadai: How can we expand $C = 3x(x - 3) - (x + 7)(4x - 2)$ without losing the sign in front of the second product, and then check the result?

R1 or T, error:

$$3x^2 - 9x - 4x^2 + 26x - 14 = -x^2 + 17x - 14.$$

R2, safe method:

$$\begin{aligned} 3x^2 - 9x - (4x^2 + 26x - 14) \\ = 3x^2 - 9x - 4x^2 - 26x + 14 \\ = -x^2 - 35x + 14. \end{aligned}$$

Possible R3:

$$3x(x - 3) + (-x - 7)(4x - 2) = -x^2 - 35x + 14.$$

$x = 1$: initial -22 ; R1 2; R2 -22 .

CENTRE: varied proof then program

S1 proof restated, then:

$$\begin{aligned} k &= a + b; k(c - d) = kc - kd \\ &= (a + b)c - (a + b)d \\ &= ac + bc - (ad + bd) \\ &= ac - ad + bc - bd. \end{aligned}$$

Program 5.3:

$$\begin{aligned} P(1) &= 3; P(2) = 5. \\ P(x) &= (x + 1)^2 - x^2 \\ &= (x + 1)(x + 1) - x^2 \\ &= x^2 + x + x + 1 - x^2 = 2x + 1. \end{aligned}$$

If x is an integer, $2x + 1$ is odd.

RIGHT: matome

Subtracting a group:

$$A - B = A + (-1)B.$$

I expand B inside brackets, then I distribute -1 to each of its terms; I simplify at the end.

A discrepant value refutes a general equality. A matching value only spot-checks it; the proof comes from justified equalities.

Trials on a program suggest a pattern; algebra establishes the result for all numbers in the domain.

Phase 5 (44–52): Protected matome, formulated then copied

Have pupils state the procedure and the logical status of each check before displaying the written summary. The varied proof, the tested error and the retained safe method remain visible during eight minutes of copying.

TO COPY INTO THE NOTEBOOK (word for word)

Subtracting a group amounts to multiplying it by the factor -1 : $A - B = A + (-1)B$. I expand the group B inside brackets, then I distribute -1 to each of its terms; I simplify at the end.

A discrepant value refutes an equality claimed to be general. A matching value does not prove it: it only checks it for that single value. The proof relies on a chain of justified equalities.

A calculation program translates into an expression $P(x)$. Trials make it possible to conjecture; algebra establishes the result for all numbers in the domain.

Phase 6 (52–55): Exit check

Expand $4(3x - 2) - (2x - 5)$ keeping the intermediate brackets. Expected: $12x - 8 - (2x - 5) = 12x - 8 - 2x + 5 = 10x - 3$.

Observe the scope of the minus sign and the simplification separately. This item is analogous to the test, but does not reproduce any of its questions.

Homework, capped workload. The family box shows the full R prescription: 1.4 b) and d). M track: reread the written summary and 1.4 C; do the five S2 evening drills; do only 1.4 b) with a check at $x = 1$, then stop before d). H track, optional: write up 1.5 or redo C using the method $-(x + 7) = -x - 7$. The brevet (French national exam at the end of Year 10) question 1 is not assigned: its factorisations using notable identities will only be practised in S7; it is saved for a later review.

Read: today's written summary and the worked solution for expression C in 1.4.

Evening drills: “Evening of S2” block, upside-down answers to be checked after trying the questions.

Do: 1.4 b) and d), checking b) with $x = 1$.

Optional: write up 1.5 or redo C by replacing $-(x + 7)$ with $-x - 7$.

The brevet question 1 waits until factorisation has been taught.

If running late. Drop P3 first and then solving “obtain 15”. The checked recap, the Kadai investigation, the complete safe method, the matome and the exit check cannot be cut. The calculation program may stop at $P(x) = 2x + 1$.

Success criteria... on the exit check, the pupil writes the brackets $-(2x - 5)$ before distributing and obtains $10x - 3$; they can explain that a numerical discrepancy refutes whereas an isolated agreement does not prove.

Teaching note: Complete lines of reasoning, genuinely produced and preserved, are compared. Here, the observed error or the explicitly marked T suggestion is set alongside the safe method; never two lines from the same script. So that the structure of the reasoning remains visible, the subtracted group, the varied proof and the logical status of the check are kept on the board until the matome. Timing and the calculation program adjust to the work collected.

S3. From the table to the proof: $(a + b)^2$ (format A, 55 min)

Pupil objective. Distinguish data, conjecture and proof; explain the origin of the term $2ab$ through two area readings, then establish $(a + b)^2 = a^2 + 2ab + b^2$ for all real numbers using the double distributive property. **Skills:** investigate, represent, reason, communicate.

Materials. Course notes, introductory activity and part 3; result from S2 $(x + 1)^2 = x^2 + 2x + 1$; exercise sheet 2.1 a), b), h); mini-whiteboards; blank squares of symbolic side $a + b$.

Phase 1 (0–7): Reactivate the data and formulate the Kadai

Copy the four rows of the table prepared at home: the gaps are 40, 24, 10 and 140, each equal to $2ab$. Next recall, without redoing the calculation, the result built in S2: $(x + 1)^2 = x^2 + 2x + 1$, the case where $b = 1$. Collect the actual formulations R1–Rk of the conjecture without correcting them word for word.

Q1. “What do these four rows and the case from S2 allow us to state? What do they not yet allow us to state for all real numbers?”, Expected: they make $(a + b)^2 = a^2 + 2ab + b^2$ plausible, but a finite list of trials does not prove it. Then write the shared Kadai: “The table and S2 suggest $(a + b)^2 = a^2 + 2ab + b^2$. Why does the term $2ab$ appear, and how can we prove this equality for all real numbers a, b ?” There is no founding vote: the prepared activity has already refuted the alleged identity $(a + b)^2 = a^2 + b^2$.

Phase 2 (7–18). Investigation: explain without choosing values

Give everyone a blank square whose side is stated to be $a + b$. Individual investigation 5 min, then comparison in pairs 6 min. Instructions: “Cut up or annotate the square to explain the term $2ab$ exactly. If you can, suggest another reading of the same square or a proof without a diagram.” The teacher circulates, notes the actual answers and selects complete lines of reasoning; they hand out neither a fill-in-the-blanks formula nor a pre-labelled dissection.

Anticipated productions as functions, not as promised copies.

P1. The table leads to the conjecture “gap = $2ab$ ”, explicitly recognised as unproven;

P2. four pieces: $a^2 + ab + ab + b^2$;

P3. contingent reading using two strips: $a^2 + ab + b(a + b)$;

P4. algebraic proof: $(a + b)(a + b) = a^2 + ab + ba + b^2$. A missing production is not fabricated. If P2 is missing after public prompts, the teacher builds the dissection with the class and marks it T. If P3 is missing, it is simply omitted. If no pupil’s work contains all of P4, the chain is rebuilt collectively from the double distributive property of S1.

Phase 3 (18–35). Neriage: compare representations, then generalise

Order P1, P2, P3 if it exists, then P4. Ask another pupil to rephrase each production before any validation. On P2, have them justify the lengths and check that the four pieces cover the square exactly. On P3, have them delimit the strip ab and the strip $b(a + b)$ to avoid a hidden overlap.

Q2. If P3 exists: “In the two drawings, what changes and what remains invariant?”, Expected: the dissection and the intermediate expression change; the total area and the two cross products remain. The equalities $a^2 + ab + ab + b^2 = a^2 + ab + b(a + b) = a^2 + 2ab + b^2$ come from the distributive property and the equality of areas.

Q3. “Are the drawings enough when $a = -3$ and $b = 1$? Which proof covers the domain required by the Kadai?”, Expected: the represented lengths are non-negative. For all real numbers, the algebraic chain establishes the identity; $ab + ba = 2ab$ by commutativity of multiplication followed by addition of two equal terms.

IDEAL BOARD (final state): to project or reproduce

LEFT: data and Kadai

Prepared data:

a	b	$(a+b)^2 - (a^2 + b^2)$	$2ab$
10	2	40	40
3	4	24	24
5	1	10	10
7	10	140	140

S2: $(x+1)^2 = x^2 + 2x + 1$.

These data suggest; they do not prove.

Kadai: The table and S2 suggest $(a+b)^2 = a^2 + 2ab + b^2$. Why does the term $2ab$ appear, and how can we prove this equality for all real numbers a, b ?

CENTRE: two readings of the same square

P2, four pieces:

$$a^2 + ab + ab + b^2.$$

Two distinct rectangles explain the coefficient 2.

P3 if it exists, two strips:

$$a^2 + ab + b(a+b)$$

$$= a^2 + ab + ab + b^2.$$

For $a, b \geq 0$, the pieces cover the large square: their sum is $(a+b)^2$.

RIGHT: general proof and matome

P4, all real numbers:

$$(a+b)^2 = (a+b)(a+b)$$

$$= a^2 + ab + ba + b^2$$

$$= a^2 + 2ab + b^2.$$

Matome:

$$(a+b)^2 = a^2 + 2ab + b^2$$

Areas explain $2ab$; the double distributive property proves the identity for all real numbers.

Phase 4 (35–44): Protected matome, formulated then copied

The universal statement “for all real numbers a, b , $(a+b)^2 = a^2 + b^2$ ” is false: for $a = 3$, $b = 4$, we get $49 \neq 25$. This counterexample does not mean that the equality fails for every pair: it is true, for example, when $b = 0$.

First have pupils formulate the distinction between the role of the table, that of the areas and that of algebra. The Kadai, one area reading and the algebraic chain remain simultaneously visible for nine minutes. Then display the written summary.

TO COPY INTO THE NOTEBOOK (word for word)

For all real numbers a and b ,

$$(a+b)^2 = a^2 + 2ab + b^2.$$

The term $2ab$ is the double product: on the square of side $a+b$ split into four pieces, it corresponds to the two rectangles of area ab ; in the calculation, it comes from the two cross products ab and ba . Areas explain the equality for non-negative lengths; the double distributive property proves it for all real numbers. An identity is an equality that is true for all permitted values of the letters.

Phase 5 (44–52): Targeted practice on the single identity of S3

Do exercise sheet 2.1 a), b), h) only. For each expression, write the two blocks before the formula: a) $a = x$, $b = 3$; b) $a = x$, $b = 10$; h) $a = n$, $b = 1$. Here, the letter a in the identity denotes a role and may be replaced by x or n .

Quick answers (teacher): a) $(x+3)^2 = x^2 + 6x + 9$ b) $(x+10)^2 = x^2 + 20x + 100$ h) $(n+1)^2 = n^2 + 2n + 1$.

Phase 6 (52–55): Exit check

Mini-whiteboard: expand $(2x+3)^2$ by first writing $a = 2x$, $b = 3$. Expected: $(2x)^2 + 2 \times 2x \times 3 + 3^2 = 4x^2 + 12x + 9$. Examine the square of the coefficient and the double product separately.

Differentiation. Struggling pupils: diagram P2 already divided, lengths a and b highlighted in colour, then template $(\dots + \dots)^2 = (\dots)^2 + 2 \times \dots \times \dots + (\dots)^2$ for the first item only. Fast finishers: 2.2 a), then write in one sentence the two-strip reading if it was not shared publicly.

Homework, capped workload. The standard family block carries the full R prescription: 2.1 a), b), h), then 2.2 a). Track M: re-read the written summary and the geometric proof of $(a+b)^2$; learn this single identity and the definition of an identity; do the five Gammes du soir (evening drills) S3; do only 2.1 a), b), h), then stop before 2.2 a). Track H, optional: draw a second decomposition of the square of side $a+b$ and write the corresponding area expression. The part 3 microtest, which already uses all three identities, only becomes available after S4. Topic 4 from the brevet (French national exam at the end of Year 10), 2.6 and clever calculation wait until at least S5.

Read: course notes p. 10, area proof of the square of a sum.

Learn by heart: $(a + b)^2 = a^2 + 2ab + b^2$; be able to explain the origin of $2ab$ and define an identity.

Evening drills: “Evening of S3” block, upside-down answers to be checked after trying the questions.

Do: 2.1 a), b), h), then 2.2 a).

Optional: suggest another dissection of the square of side $a + b$ and write the corresponding area expression.

The quick questions in the course notes p. 15 and brevet topic 4 wait for the next identities; exercise 2.6 waits until lesson S5.

End of week 1: light weekend. No full brevet topic is set before factorising has been taught. The variant below replaces the “Evening drills” and “Do” lines of the standard block: thirty minutes of review/error log if an error is designated, otherwise rest. The topic weekend comes at the end of week 3 (sheet S11), once choosing the right form has been seen in class and the worked solutions are open.

End-of-week-1 light weekend variant (ready to paste).

Read: course notes p. 10, area proof of the square of a sum.

Learn by heart: $(a + b)^2 = a^2 + 2ab + b^2$; be able to explain the origin of $2ab$ and define an identity.

Rework: an error already spotted in the error log, if designated by the teacher.

No other work is added; if no review is designated, rest.

If running late. Drop P3 first, then reduce practice to 2.1 a) and h). The starting data, a complete visual justification, the algebraic proof, the nine-minute matome and the exit check are incompressible. The other two identities belong to S4 by default, not as a fallback solution.

Success if... on the exit check, the pupil identifies $a = 2x$ and $b = 3$, obtains $4x^2 + 12x + 9$ and can point out the origin of the term $12x$; orally, they distinguish “trials suggest”, “areas explain for lengths” and “algebra proves for all real numbers”.

Teaching note: A single problem, investigation time, then answers actually produced, selected and connected; P3 is omitted if it does not appear and any essential input is marked T. On the board: stable zones, progressive writing and visible architecture of the reasoning; only inscriptions that have fulfilled their role may be erased. Listen to and observe pupils’ ideas without making premature judgements about their correctness; have a classmate’s idea rephrased in the pupil’s own words rather than repeating it for them; do not let the class focus solely on the right answers of a few pupils.

S4: One same square, two remaining identities (format A, 55 min)

Pupil objective. From a single partitioned square, build $(a - b)^2 = a^2 - 2ab + b^2$ and $a^2 - b^2 = (a + b)(a - b)$; distinguish their forms and the domains of the geometric and algebraic proofs. Reinvest the definition of an identity on three carefully chosen statements. **Skills:** investigate, represent, reason, communicate.

Materials. One sheet per pupil with two unlabelled copies of the same square of side a , partitioned by the lengths $a - b$ and b ; one pair of cut-out pieces per pair of pupils; course notes closed during the investigation, then part 3; exercise sheet 2.4 b), f), g); mini-whiteboards.

Phase 1 (0–6): Reactivate S3, spark the problem

On mini-whiteboards, revisit a single item actually set in S3: $(x + 10)^2 = x^2 + 20x + 100$. Have pupils name the two cross products, then write alongside $(a - b)^2$ and $a^2 - b^2$ without expanding them.

Q1. “Do these last two expressions mean the same thing? If we replace the $+$ sign from S3 with a $-$ sign, what can we predict and what remains to be proved?”, Expected: the square of a difference and the difference of two squares are two distinct expressions; we may conjecture a negative double product, but analogy does not constitute a proof. Display the Kadai: “How can the same dissected square explain the two missing identities, and what proof makes them true for all real numbers?”

Phase 2 (6–18): Individual then pair investigation on the master drawing

Specify only $0 < b < a$. On the first drawing, ask for the area of the square of side $a - b$ in two ways. On the second, ask for the area of the remaining surface after removing the square b^2 , then allow cutting, rotating and reassembling. Individual investigation 5 min, comparison in pairs 7 min. Written instructions: “Name the lengths and areas; write a chain of equalities; indicate what is cut out or moved. If you can, add a proof without a diagram.”

Anticipated productions as functions, not as promised copies.

P1. Partition into four pieces: $a^2 = (a - b)^2 + 2b(a - b) + b^2$, then isolating the square;

P2. contingent reading using two full strips: $a^2 - ab - ab + b^2$;

P3. the remaining L-shape equals $a(a - b) + b(a - b)$, then reassembles into a rectangle with sides $a + b$ and $a - b$;

P4. double distributive property on $(a - b)(a - b)$ and $(a + b)(a - b)$. The teacher notes the actual answers R1–Rk and chooses their order. If P2 is missing, it is omitted. If P3 does not go as far as the rectangle, the rotation is built with the class and marked T. If no pupil’s work contains all of P4, the two chains are rebuilt collectively from S1 and marked T.

Phase 3 (18–34). Neriage: two regions, then extending the domain

Have another pupil rephrase each production. For P1, check the four areas and isolate $(a - b)^2$ without skipping the expansion of $-2b(a - b)$. For P2, have pupils explain why b^2 is added back after being subtracted twice. For P3, name the two rectangles before factorising. Finally, compare the domain $0 < b < a$ of the drawings with the domain of P4.

Q2. “In P1 and P2, which same area is being calculated? Why are the expressions $a^2 - 2b(a - b) - b^2$ and $a^2 - 2ab + b^2$ equal?”, Expected: the square of side $a - b$; $-2b(a - b) = -2ab + 2b^2$, then $2b^2 - b^2 = b^2$.

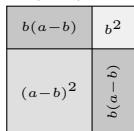
Q3. “In P3, what are the exact dimensions of the two pieces? Which algebraic property do we read when $a(a - b) + b(a - b)$ becomes $(a + b)(a - b)$?”, Expected: rectangles $a \times (a - b)$ and $b \times (a - b)$; factoring out $a - b$, which is the distributive property read in reverse.

Q4. “Does the drawing represent $a = -2$ and $b = 5$? Which chains nevertheless prove both equalities for these values and for all real numbers?”, Expected: no; lengths require $0 < b < a$. The double distributive property establishes both identities for all real numbers.

IDEAL BOARD (final state): to project or reproduce

LEFT: master drawing and Kadai

$0 < b < a$ for the diagram.



Kadai: How can the same dissected square explain the two missing identities, and what proof makes them true for all real numbers?

CENTRE: two regions of the same square

P1, the inner square:

$$a^2 = (a-b)^2 + 2b(a-b) + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2.$$

P2 if it exists, strips:

$$(a-b)^2 = a^2 - ab - ab + b^2.$$

P3, the L-shape without b^2 :

$$a^2 - b^2 = a(a-b) + b(a-b)$$

$$= (a+b)(a-b).$$

RIGHT: general proof and matome

P4, all real numbers:

$$(a-b)^2 = (a-b)(a-b)$$

$$= a^2 - ab - ba + b^2$$

$$= a^2 - 2ab + b^2,$$

$$(a+b)(a-b) = a^2 - ab + ba - b^2 = a^2 - b^2.$$

Matome:

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a+b)(a-b)$$

Square of a difference $\neq$ difference of two squares.

Logical contrast (phase 5): 2.4 b) false; 2.4 f) false: the pair (3, 4) refutes “for all”; 2.4 g) true, the witness $b = 0$ suffices.

Phase 4 (34–42): Protected matome, formulated then copied

Keep the master drawing, P1, P3 and the two P4 chains simultaneously visible. Have pupils formulate the difference between the two expressions, then display the written summary.

TO COPY INTO THE NOTEBOOK (word for word)

For all real numbers a and b ,

$$(a-b)^2 = a^2 - 2ab + b^2 \quad \text{and} \quad a^2 - b^2 = (a+b)(a-b).$$

Do not confuse the square of a difference, which expands into three terms, with the difference of two squares, which is written as a product. The two dissections of the square of side a explain them when $0 < b < a$; the double distributive property proves them for all real numbers.

Phase 5 (42–50). Three useful votes: 2.4 b), f), g)

The word identity was defined in S3: here it is reinvested, not introduced as new. For each of the three items: read, vote, record the distribution, collect a justification, then vote again. b) is false: for 5 and 2, $(5-2)^2 = 9$ whereas $5^2 - 2^2 = 21$. f) is false: $(3+4)^2 = 49 \neq 25$. g) is true: the witness $b = 0$ suffices to establish “there exists”. An existence witness does not turn f) into an identity.

Q5. “Why does the same choice $b = 0$ not prove f), yet prove g)?”, Expected: f) is about all pairs; g) requires just one pair. A counterexample suffices to refute f), whereas a witness suffices to establish g).

Phase 6 (50–55): Contrasted exit check

Expand the two expressions side by side on mini-whiteboards: $(3x-2)^2$ and $(3x+2)(3x-2)$. Require the name of the structure before calculating.

Quick answers (teacher): $(3x-2)^2 = 9x^2 - 12x + 4$; $(3x+2)(3x-2) = 9x^2 - 4$. The term $-12x$ immediately distinguishes the two forms.

Differentiation. Struggling pupils: drawing with lengths $a-b$ and b already highlighted, then P1 to complete; on the exit check, only one expression assigned, the other being explained by their partner. Fast finishers: produce P4 without a diagram, then 2.2 f) on the auxiliary strip.

Homework, capped workload. The family block carries the full R prescription: finish 2.1, do 2.2 c), e), g), then justify 2.4 a), c), d), e). Track M: re-read the two geometric proofs and the written summary (course notes p. 12); learn the three identities in the form written in the course notes p. 11; do the five evening drills S4; do 2.1 c), e), i), then finish the drill with d), f), g) and stop before 2.2. Track H, optional: add 2.5 a)–b). The part 3 microtest becomes available for self-assessment, without being added to the homework. Exercise 2.3 and the factorisation method wait until S6; brevet 4 waits until at least S5.

Read: course notes p. 12, geometric proofs of the square of a difference and the difference of two squares.

Learn by heart: course notes p. 11, the three notable identities.

Evening drills: “Evening of S4” block, upside-down answers to be checked after trying the questions.

Do: 2.1 c), e), i); 2.2 c), e), g); 2.4 a), c), d), e).

Finish: drill 2.1 with d), f) and g).

Optional: 2.5 a) and b), “With fractions”.

The quick questions in the course notes p. 15 are available; 2.3 waits for factorising and brevet topic 4 for clever calculation.

If running late. Drop P2 first, then limit the votes to the pair f)–g). P1, P3, the two P4 chains, the matome and at least one exit expression are incompressible; no unexplained technical exercise is automatically pushed into homework.

Success if... on the exit check, the pupil obtains $9x^2 - 12x + 4$ and $9x^2 - 4$ without confusing the structures; orally, they can state that the diagrams assume $0 < b < a$, that algebra covers all real numbers and that the witness $b = 0$ proves an existence, not an identity.

Teaching note: A central problem, substantial investigation time, then actually available answers selected, ordered and connected. A master drawing and stable zones make the transformations visible; P1, P3 and P4 remain simultaneously readable right up to the matome. Do not signal agreement or rephrase a good idea yourself; have pupils rephrase and compare ideas without premature judgement; any essential input is marked T. The pair f)–g) varies the quantifier while keeping the same equality, which precisely isolates the logical status of an example.

S5. Clever calculation: the reward (format B, 55 min)

Pupil objective. Distinguish a valid transformation from a genuinely economical one; choose between a notable identity and the distributive property; check the plausibility of a result. **Skills:** calculate, investigate, reason, communicate.

Materials. Course notes part 5; exercise sheet 4.1; cards $A = 10,1 \times 9,9$ and $B = 38 \times 998$; 4.4 d; 4.5 a, c; timer; mini-whiteboards.

Cultural reserve, outside the slot: Gauss and 5 050

The school version often told of the Gauss episode leads to pairing the sum $1 + 2 + \dots + 100$ with its reversed form, and then obtaining 5 050. This problem is not added to the 55-minute scenario below: it would change its Kadai, investigation time and matome. Keep it for an extra lesson equipped with its own Bansho. The exact generalisation is: for every positive integer n , $1 + 2 + \dots + n = \frac{n(n+1)}{2}$.

Phase 1 (0–8). 4.1: five fixed minutes, then self-marking

Announce: “zero column multiplication; every written equality must be justified”. Exactly five minutes, then everyone stops. Display the answers and allow three minutes for self-marking; discuss only c) and g). Record two anonymous class indicators: number of pupils with at least six correct answers, then eight out of eight. Repeat the same measurement, under the same conditions, in S10; we compare accuracy over a fixed time, not an ill-defined speed.

Quick answers (teacher): a) 10 201 b) 9 801 c) 9 996 d) 9 991 e) 441 f) 361 g) 1 225 h) 1 002 001. For c): centre 100, gap 2. For g): $(30 + 5)^2 = 900 + 300 + 25$.

Phase 2 (8–11): Two calculations, then the Kadai

Write only $A = 10,1 \times 9,9$ and $B = 38 \times 998$. Kadai: “Is a notable identity always the best tool whenever it is possible? How do we choose a transformation that genuinely reduces the calculation?” Shared instructions: for each calculation, suggest an exact rewriting, name the tool and specify what remains to be calculated.

Phase 3 (11–20): Individual then pair investigation

Four minutes alone, five minutes in pairs. While circulating, note the actual answers R1–Rk and, for each route, the

remaining operations. Do not announce that an identity is “impossible” for B : any product of two real numbers can be centred around their mean; the question here concerns the economy achieved.

Q1. “Does your rewriting give the exact same number? Which operations remain after this rewriting?”, Goal: separate the two criteria before pooling ideas.

Phase 4 (20–33). Neriage: P1, P2 or T, then P3

P1: $A = (10 + 0,1)(10 - 0,1) = 100 - 0,01 = 99,99$. Have pupils name the centre 10 and the gap 0,1. P2: for B , the route $(518 - 480)(518 + 480) = 518^2 - 480^2$ is exact since 518 is the mean of 38 and 998. If it does not appear, rebuild it publicly and mark it T. However, it is costly. P3: $B = 38(1000 - 2) = 38\,000 - 76 = 37\,924$; here the distributive property is more economical.

Q2. “Are P2 and P3 both true? Which one genuinely reduced the work, and where can we see this?”, Expected: both equalities are valid; P2 requires two large squares, P3 only two easy products and a subtraction.

Phase 5 (33–40). Transfer P4: the zero result

Set $2026^2 - 2025^2 - 4051$. Two minutes alone, two minutes in pairs, then whole-class discussion. Keep the chain

$$2026^2 - 2025^2 - 4051 = (2026 + 2025)(2026 - 2025) - 4051 = 4051 - 4051 = 0.$$

Q3. “Which sub-calculation needed to be transformed before any approximation? Why is the identity advantageous here?”, Expected: the gap is 1, so the resulting product is immediate.

Phase 6 (40–46). 4.5 a, c: check two statements

Present a) as an alleged identity, hence a statement for every real number x , and c) as a numerical equality. Vote, justification, re-vote. a) is refuted by $x = 1$ ($121 \neq 101$) or by the missing double product $20x$; the equality is nevertheless true for one value, $x = 0$. c) is refuted by $(50 - 3)(50 + 3) = 2500 - 9 = 2491$, or by the units digit.

Q4. “Does a counterexample to a) prove that the equality fails for every value of x ”, Expected: no; it only refutes the statement ‘for all x ’.

IDEAL BOARD (final state): to project or reproduce

LEFT: data, Kadai and P1

4.1: $n(\geq 6/8) = \underline{\hspace{1cm}}$; $n(8/8) = \underline{\hspace{1cm}}$.
In S10: same set, no column multiplication, every equality justified, alone, everyone stops at the same signal.
 $A = 10,1 \times 9,9$; $B = 38 \times 998$.

Kadai: Is a notable identity always the best tool whenever it is possible? How do we choose a transformation that genuinely reduces the calculation?

P1: $A = (10 + 0,1)(10 - 0,1) = 99,99$.

CENTRE. P2–P4: validity then cost

P2 or T: $B = (518 - 480)(518 + 480) = 518^2 - 480^2$: exact, but costly.

P3: $B = 38(1000 - 2) = 37\,924$: exact and economical.

P4: $2026^2 - 2025^2 - 4051 = (4051)(1) - 4051 = 0$.

Valid does not always mean efficient.

RIGHT: matome, checks and exit

Matome: preserve equality; reduce remaining calculations; check. A round number is a clue, not a rule.

$(x + 10)^2 = x^2 + 100$ is not an identity: $x = 1$ refutes it.

$47 \times 53 = 2500 - 9 = 2491$.

Exit: $49 \times 51 = 50^2 - 1^2 = 2499$.

Phase 7 (46–52): Protected matome, formulated then copied

Keep P1, P2 and P3 simultaneously visible. Have pupils formulate three requirements: the rewriting preserves the number; it makes the remaining operations simpler; a final check remains necessary.

✏ TO COPY INTO THE NOTEBOOK (word for word)

Clever calculation. I rewrite without changing the value, then I choose the route that makes the remaining calculations simplest. An identity can be exact without being advantageous: $38 \times 998 = 518^2 - 480^2$, but $38(1000 - 2) = 37\,924$ is far more economical. A round number is a clue, not a rule.

Checking. Before handing in my work, I check the structure, the sign, the units digit or a test value. A counterexample refutes a “for all” statement, without proving that the equality fails for every value.

Phase 8 (52–55): Individual exit check

Mini-whiteboard: 49×51 . Require the transformation, the name of the tool and the result: $(50 - 1)(50 + 1) = 50^2 - 1^2 = 2499$.

Differentiation. Struggling pupils: provide the fill-in-the-blanks expressions $(10 + \square)(10 - \square)$ and $38(1000 - \square)$. Fast finishers: establish $uv = \left(\frac{u+v}{2}\right)^2 - \left(\frac{u-v}{2}\right)^2$, then explain why this general identity is only useful if the two new numbers are easy to calculate.

Homework, capped workload. The family block carries the full R prescription: 4.2 a)–f), 4.3 a)–c), then 4.5 b), d). Track M: re-read the written summary; do the five evening drills S5; finish only the items from 4.1 not completed within the five minutes; do only 4.2 a), c), e) and 4.5 b), d), then stop before 4.3. Track H, optional: add 4.4 a) (the only item in 4.4 not done in class) or challenge 8.1, as preferred. Exercise 3.5 and the general factorisation method wait until S6; no new concept is sent home in advance.

Read: course notes p. 20 (part 5) and p. 21, clever mental calculation method (models C1 and C2).

Evening drills: “Evening of S5” block, upside-down answers to be checked after trying the questions.

Do: 4.2 a) to f), 4.3 a) to c), then 4.5 b), d).

Finish: only the questions from 4.1 not obtained in class.

Optional: 4.4 a), “Champions: the big league”, the only calculation in this exercise not done in class.

Optional, as an alternative to the previous one: challenge 8.1, “An almost prime square”.

The general factorisation method for 3.5 will be built in the next lesson.

If running late. Display the answers to 4.1 without commenting on all of them, then omit 4.5 c). P1, P2 or T, P3, P4, the matome and the exit check remain incompressible: without the “valid / efficient” comparison, the Kadai is not resolved.

Success if... the pupil obtains 2499 on the exit check with centre 50; they can state that $518^2 - 480^2$ is a valid rewriting of 38×998 but less economical than $38(1000 - 2)$; the five-minute indicator is kept for the S10 comparison.

Teaching note: [S] A teaching tradition committed to fundamentals aims for a balance between mastering basic knowledge and skills and the ability to CHOOSE an efficient method, rather than speed alone; a test focusing specifically on algebra (grouping, completing the square, factorising), 38 questions in 10 minutes, distinguishes speed from accuracy: this is the exact regime of 4.1, and the reason for recording accuracy over a fixed time rather than a speed ranking. [S] On variation: vary NON-ESSENTIAL features while keeping the essential invariant, so that the latter becomes discernible; the two forms of variation work together. [T] Here, A and B vary the cost while maintaining algebraic validity; the neriage therefore separates two judgements that are often confused. [D] If pupils still favour the chapter title over the actual cost, revisit in S6 a pair “valid but long identity / short distributive property”. The compared routes are kept together on the board, and each idea is rephrased without premature judgement or fictitious attribution.

S6. Factorising: the repeated block, then the photographer (format A, 55 min)

Pupil objective. Transform a sum into a product using a common factor; distinguish a valid factorisation from a fully completed extraction; justify the equality for all values of x ; then open the route to identities. **Skills:** calculate, reason, communicate.

Materials. Course notes part 4; exercise sheet 3.1 b), 3.2 B–D, 2.3 a) first expression; mini-whiteboards. No worked solutions for 3.5 at the start: S5 explicitly deferred it.

Phase 1 (0–6): Reactivate the forward direction, then reverse the arrow

Write $3x(4x - 5)$ and ask for its expansion: $3x(4x - 5) = 12x^2 - 15x$. Then reverse the arrow, without giving a procedure: $12x^2 - 15x \rightarrow 3x(4x - 5)$.

Q1. “What changes when we read this equality the other way round?”, Goal: create the need to look for a structure. Expected: we must recognise what multiplies both terms. Minimal institutionalisation: to factorise is to rewrite a sum as an equal product; it is not a single unique inverse operation.

Phase 2 (6–10): Present a family and set the Kadai

Write, without circling or colouring the factors:

$$F_1 = 12x^2 - 15x, \quad F_2 = (x + 5)(4x - 2) - (x + 5)(9x + 1), \quad F_3 = (4x + 5)^2 - 5(4x + 5).$$

Kadai: How can we transform each of the three sums into a product without losing a term or a sign? Which repeated block is common to all three, and how can we prove the equality for all values of x ?

Phase 3 (10–21). Investigation: five minutes alone, then six minutes in pairs

Written instruction: “For each expression, propose a product; circle the block you saw repeated; prepare one line that justifies the equality.” During the investigation, note actual pupil work without announcing $3x$, without naming the common bracket and without fabricating an error. Label the selected work C1, C2, etc., without names or initials.

Phase 4 (21–34). Neriage: valid, more complete, compound, hidden

Select in this order, adapting C1–C4 to the work that actually appeared.

Q2. “ $F_1 = 3(4x^2 - 5x)$: is this already a product? Can we still extract something in common?”, Expected: the first form is a valid factorisation; x remains in common, so $F_1 = 3x(4x - 5)$ extracts all visible common factors. Do not call the first one wrong.

Q3. “In F_2 , what must $(x + 5)$ multiply to give back each of the two terms?”, Expected: $F_2 = (x + 5)[(4x - 2) - (9x + 1)] = (x + 5)(-5x - 3)$. If the error $-9x + 1$ appears, cross it out after refuting it; otherwise, do not invent it.

Q4. “How can we make the repeated block visible in F_3 ?”, Expected: $(4x + 5)^2 = (4x + 5)(4x + 5)$, then $F_3 = (4x + 5)[(4x + 5) - 5] = 4x(4x + 5)$.

Q5. “Which line proves each equality for all values of x ? Would agreement obtained only at $x = 0$ be enough?”, Expected: re-expanding using the distributive property gives back the initial expression for all x . A disagreement for one value refutes; an agreement for a single value does not prove an identity.

Phase 5 (34–41): Protected matome on the common factor

Have pupils formulate the invariant $kA + kB = k(A + B)$ and $kA - kB = k(A - B)$: the factor k must multiply all terms; the square bracket contains exactly what is left to rebuild each term. Explicitly compare “valid product” and “further extraction” on P1–P2.

Phase 6 (41–48): Only then open a second route

Write the preliminary rule: “first look for a factor common to all terms”. In its absence, introduce the photographer’s three questions. Use the first item of exercise sheet 2.3, $x^2 + 10x + 25 = (x + 5)^2$, then the contrast $x^2 - 8100 = (x + 90)(x - 90)$ (a value absent from the exercise sheet, course notes, worked solutions, multiple-choice quiz and

test: $8100 = 90^2$ is read from $81 = 9^2$). Announce them as teacher input T, and not as pupil work if they were not investigated.

IDEAL BOARD (final state): to project or reproduce

LEFT: shared problem

$$3x(4x - 5) = 12x^2 - 15x \leftarrow \text{factorise}$$

Kadai: How can we transform each of the three sums into a product without losing a term or a sign? Which repeated block is common to all three, and how can we prove the equality for all values of x ?

$$F_1 = 12x^2 - 15x$$

$$F_2 = (x + 5)(4x - 2) - (x + 5)(9x + 1)$$

$$F_3 = (4x + 5)^2 - 5(4x + 5)$$

CENTRE: compared work

P1: $F_1 = 3(4x^2 - 5x)$: valid product; x remains in common.

P2: $F_1 = 3x(4x - 5)$: further extraction.

P3: $F_2 = (x + 5)[(4x - 2) - (9x + 1)] = (x + 5)(-5x - 3)$.

P4: $F_3 = (4x + 5)[(4x + 5) - 5] = 4x(4x + 5)$.

RIGHT: proof and second route

$kA + kB = k(A + B)$ and $kA - kB = k(A - B)$, by the distributive property. Re-expanding proves the equality for all x .

A test value: disagreement $\Rightarrow$ false; agreement $\nRightarrow$ proof.

Before identities: common factor? Otherwise: number of terms; squares; double product.

$$x^2 + 10x + 25 = (x + 5)^2$$

$$x^2 - 8100 = (x + 90)(x - 90)$$

Phase 7 (48–53): Short written summary, formulated then copied

✍ TO COPY INTO THE NOTEBOOK (word for word)

To factorise a sum is to rewrite it as an equal product. For a common factor k , I write each term as $k \times \dots$, then I use $kA + kB = k(A + B)$ and $kA - kB = k(A - B)$. A factorisation can be correct without having extracted all visible common factors. Re-expanding proves the equality for all x ; agreement for a single value does not prove it. I look for a common factor first; otherwise, I recognise an identity from the structure.

Phase 8 (53–55): Two-route individual exit

Mini-whiteboard: factorise $7x - 35$, then $x^2 - 36$. Require the name of the route before calculating: $7(x - 5)$; $(x + 6)(x - 6)$. Have pupils mentally re-expand one of the two answers.

Differentiation. Struggling pupils: give the intermediate forms $12x^2 = 3x \times \square$ and $(4x + 5)^2 = (4x + 5) \times \square$. Fast pupils: do 3.2 C, then compare two equivalent factorisations if an additional numerical factor can be extracted. Do not anticipate 3.3: its traps form the problem of S7.

Homework, capped workload. The family block carries the full R prescription: the first two expressions of 2.3 a), 3.2 A and all six lines of 3.5. Track M: reread the written summary; do the five S6 evening drills; finish only the items of 3.1 not completed in class; do the first two expressions of 2.3 a); complete only the first four lines of 3.5, then stop before 3.2 A and the last two lines. Track H, optional: add 8.2. No item from 3.3 is assigned before its investigation in S7.

Read: course notes pp. 15 to 18, definition of factorising and photographer's method.

Evening drills: "S6 evening" block, upside-down answers to check after trying.

Do: 2.3 a), first two expressions; 3.2 A; 3.5, all six lines.

Finish: only the questions of 3.1 not completed in class.

Optional: 8.2, "Proof without words".

The traps in 3.3 will be investigated in class.

If running late. Reduce the identity opening to the single example $x^2 + 10x + 25$ and postpone $x^2 - 8100$ to the start of S7. Never cut the investigation on F_1, F_2, F_3 , the comparison P1–P4, the proof by re-expanding or the distinction "single-value agreement / general proof".

Success criteria... the pupil factorises both exit expressions while naming two different routes; accepts $3(4x^2 - 5x)$ as a valid product while knowing how to continue the extraction; can explain why agreement at $x = 0$ does not prove an equality for all x .

Teaching note: [S] The variation "visible factor / whole bracket / factor revealed by a square" keeps the same invariant while shifting what masks it: this is conceptual variation, varying non-essential features to make the essential discernible. [S] Accepting a range of degrees of completion, rather than only the answer closest to the teacher's: the teacher accepts and

values pupil ideas spanning a wide range of sophistication. [T] Keeping P1 and P2 visible simultaneously makes it possible to discuss two degrees of completion without turning a true answer into an error. [D] If pupils look for an identity before the common factor, revisit the same item in S7 via two paths and compare their cost. Numerical testing is kept as a tool for refutation, never as a proof of an identity.

S7: Classifying four borderline cases, then judging Louise's paper (format B, 55 min)

Pupil objective. Formulate an accurate and justified verdict on four expressions; compare two valid routes; distinguish a test that refutes from an algebraic calculation that proves. **Skills:** calculate, reason, communicate.

Materials. Four cards from exercise sheet 3.3 (f, i, h, j); exercise sheet 3.6; photographer's poster; mini-whiteboards; "clues / verdict / reason" grid for pupils who need it.

Phase 1 (0–6): Bounded review of S6's back-and-forth

Go over only two contrasting lines of 3.5: $(x + 30)^2 = x^2 + 60x + 900$ and $x^2 - 12x + 36 = (x - 6)^2$. Have pupils name the direction of the work. In both cases, re-expanding establishes the equality for all x ; substitution would only check a single value.

Q1. "If both expressions give the same value for $x = 0$, is the identity proved?", Expected: no. Agreement for a single value proves nothing general; a chain of equalities or re-expanding is required. A disagreement, however, would be enough to refute.

Phase 2 (6–9): Handing over the same problem on four cards

Display without comment: A $64 - 80x + 25x^2$; B $16x^2 - 144$; C $x^2 - 13$; D $x^2 + 81$. Write the Kadai: "What accurate verdict can we justify for each card, and from what clues? When several routes exist, how can we compare them?" Forbid bare verdicts "possible / impossible": each group must give a reason and specify the scope of its answer.

Phase 3 (9–20): Individual investigation, then pair discussion

Five minutes individually, then six minutes in pairs. Each pupil fills in four lines: observed clues; verdict; first step; method of justification. Do not give $\sqrt{13}$ during the investigation. If a group concludes too quickly "not factorisable", prompt: "with what tools? in what set of numbers? what can you actually prove today?"

Anticipation.

- A ✗ the order prevents seeing the square; have them circle $64 = 8^2$, $25x^2 = (5x)^2$ and calculate $2 \times 8 \times 5x$;
- B ✓ two different products are proposed; keep both to compare accuracy and efficiency;
- C ✗ "13 is not a square, so never"; have them replace never with a verdict restricted to the available numbers;
- D ✗ improperly turning the $+$ into $-$; ask which exact identity is being invoked.

Phase 4 (20–34). Neriage: four pieces of work answering the same Kadai

Select one piece of work per function and keep them simultaneously visible. Planned order: recognition despite the order (P1); comparison of two valid routes (P2); provisional limit (P3); non-applicability of the chapter's three identities (P4).

Q2. (P1) "Does the order of the three terms change their structure? Where is the double product?", Expected: $64 - 80x + 25x^2 = (8 - 5x)^2 = (5x - 8)^2$.

Q3. (P2) "Are both products accurate? Which one brings out the common numerical factor earlier?", Expected: $(4x + 12)(4x - 12)$ and $16(x + 3)(x - 3)$ are equal to the expression; the first is accurate but unfinished: step 4 of model F3 still requires factoring out 4 from each bracket; extracting 16 first leads there more directly. Do not declare the first form wrong.

Q4. (P3–P4) "Do the two dead ends have the same status?", Expected: C awaits a new numerical object; the teacher then adds, marked T, that in Chapter 02 $13 = (\sqrt{13})^2$. For D, the $+$ sign matches none of the three identities: we leave the expression unfactorised in this chapter. Do not ask in S7 for the proof of the absence of first-degree real factors (it will come in the cross-topic block, c) ii)).

Phase 5 (34–40): Protected matome on the verdicts

First have the four lines formulated by the pupils, then stabilise: structure does not depend on order; two accurate factorisations can be compared without invalidating either one; "not yet with our numbers" and "none of the chapter's identities applies" are two distinct conclusions. Keep the four pieces of work and their reasons visible.

Phase 6 (40–49). Louise: logical transfer test

Read Louise's paper on $E = 4x^2 - 4x + 1$, then individual vote. Expand $4x(x - 1)$: the $+1$ is missing. Test $x = 1$: $1 \neq 0$, so the claim is refuted. Have pupils recognise $E = (2x - 1)^2$, then re-expand: this last equality holds for all real numbers x .

Q5. “Does testing $x = 1$ prove the correct factorisation?” Expected: no. It only refutes Louise's. Re-expanding $(2x - 1)^2$ establishes the correct identity for all x .

IDEAL BOARD (final state): to project or reproduce

LEFT: Kadai and cards

Kadai: What accurate verdict can we justify for each of the four cards, and from what clues? When several routes exist, how can we compare them?

A $64 - 80x + 25x^2$

B $16x^2 - 144$

C $x^2 - 13$

D $x^2 + 81$

Clues: number of terms; squares; sign; double product; common factor.

CENTRE: P1–P4 compared

P1 A = $(8 - 5x)^2$: order has no effect on structure.

P2 B = $(4x + 12)(4x - 12)$
= $16(x + 3)(x - 3)$: complete form (F3, step 4); extracting 16 first is more direct.

P3 C: not yet with the available numbers.

T Ch. 02: $13 = (\sqrt{13})^2$.

P4 D: none of the chapter's three identities applies; leave unfactorised here.

RIGHT: transfer and matome

Louise: $E = 4x^2 - 4x + 1$ and $4x(x - 1)$.

$x = 1$: $1 \neq 0 \Rightarrow$ claim refuted.

$(2x - 1)^2 = 4x^2 - 4x + 1$ for all x .

Matome: disagreement at one value $\Rightarrow$ false; agreement at one value $\nRightarrow$ proof; re-expanding establishes the identity.

Phase 7 (49–53): Short written summary, after the comparison

TO COPY INTO THE NOTEBOOK (word for word)

To factorise, I always specify the scope of my verdict. The order of the terms does not change the structure. Two different factorisations can both be accurate; I then compare their efficiency. $x^2 - 13$ awaits the square root, whereas none of the chapter's three identities applies to $x^2 + 81$.

A value that gives two different results refutes a claimed identity. A value that gives the same result proves nothing general. To establish a factorisation for all x , I re-expand the product and recover the starting expression.

Phase 8 (53–55): Individual exit

Hand out alternately: $25x^2 - 81$ or $x^2 + 16$. Require a complete verdict and its reason. Expected: $(5x + 9)(5x - 9)$; or “none of the chapter's three identities applies, so I leave the expression unfactorised here”.

Differentiation. Struggling pupils: pre-structured “clues / verdict / reason” grid, without asking them to anticipate $\sqrt{13}$. Fast pupils: write both complete routes for B and prove their equality by re-expanding; only after the matome, do 3.4 a). Track H, optional: 3.7 a); full grouping stays in the bank.

Homework, capped workload. The family block carries the full R prescription: 3.3 a)–e), g), k), l), then 3.4 a), b). Track M: reread the written summary and Louise's paper; do the S7 evening drills; do only 3.3 a), c), g), l), then 3.4 a), and stop before the remaining items. Track H, optional: add 3.7 a). Items 3.4 c)–d), the rest of 3.7 and 5.7 stay in the bank; they are not due.

End of week 2: light weekend. Brevet 3 awaits choosing the right form (model V1, question c), covered in class in S10) and the exercise sheet worked solutions for self-marking (handed out in S9): it is placed at the end of week 3 (sheet S11). This weekend, the light variant below replaces the “Evening drills” and “Do” lines of the standard block: thirty minutes of review/error log if an error is flagged, otherwise rest. 5.7 is not added to this weekend.

Next week's flash quiz. The structure of 3.4 a), worked on by everyone, may be reused with changed numbers; formative by default, with criteria and an opportunity to retake if a mark is announced.

Read: today's written summary and Louise's paper.
 Evening drills: "S7 evening" block, upside-down answers to check after trying.
 Do: 3.3 a) to e), g), k), l), then 3.4 a) and b).
 Optional: 3.7 a), "Factorising by grouping": finish if started in class, discover otherwise.
 The weekend is light: the end-of-week instruction replaces the evening drills and exercises.

Light weekend variant (to paste in place of the "Evening drills" and "Do" lines).

Read: today's written summary and Louise's paper.
 Rework: an error already spotted in the error log, if the teacher flags it.
 No other work is added.

If running late. Cut the opening review to three minutes and Louise to seven minutes, but keep the investigation, P1–P4, the matome and the refute/prove distinction. No extra 3.4 exercise is automatically pushed to homework.

Success criteria... the pupil factorises $25x^2 - 81$, formulates for $x^2 + 16$ a verdict restricted to the chapter's identities, and explains that a single disagreement refutes whereas re-expanding proves the general equality.

Teaching note: [S] The four cards organise a controlled variation: same classification question, shifted clues and contrasting answer statuses; this is conceptual variation, which explicitly includes variation by non-examples, i.e. counterexamples delimiting the scope of a concept. [T] P1–P4 remain visible together so that the discussion focuses on their relationships rather than four successive corrections. [D] Writing with $\sqrt{13}$ is a deferred teacher input T, not required pupil work; sweeping absolute statements about real factorisation are avoided as long as they cannot be proved with the pupils' tools.

S8: From checked cases to two general proofs (format B, 55 min)

Pupil objective. Distinguish checked cases, conjecture and proof; prove that the sum of three consecutive integers is three times the middle number by comparing two choices of letter; then refute a universal variant with a counterexample. **Skills:** investigate, reason, communicate.

Materials. Course notes part 5; exercise sheet 5.3 c) and 5.4; prepared spreadsheet with a first column running from -100 to 99 ; mini-whiteboards; circulation plan R1–Rk; exit slip. The full worked solutions for the exercise sheet are not handed out before the work on Arthur prepared for S9.

Phase 1 (0–6): Fulfil the announced revisit of 5.3

Reopen only 5.3 c), solved by expanding in S2. In a side column, have pupils look for the second route:

$$P = (x + 1)^2 - x^2 = [(x + 1) + x][(x + 1) - x] = (2x + 1) \times 1 = 2x + 1.$$

Compare the two proofs: expanding is systematic; factorising brings out the sum and the difference. Both chains hold for every real number x . This revisit pays off a progression debt; it does not become a second Kadai.

Phase 2 (6–9): Handing over the Kadai

Write without any general formula: "The sum of three consecutive integers always seems to be three times the middle number. How can we prove this for all integers without being able to test them all?" Have pupils clarify the two quantities: three consecutive integers and three times the middle number. Give neither the letter nor the choice of which number it stands for.

Phase 3 (9–22): Data, conjecture, then search for a proof

From 9 to 15, calculate and write up three deliberately contrasting cases:

$$\begin{aligned} (-8) + (-7) + (-6) &= -21 = 3 \times (-7), & 7 + 8 + 9 &= 24 = 3 \times 8, \\ 199 + 200 + 201 &= 600 = 3 \times 200. \end{aligned}$$

The spreadsheet starts at -100 and produces 200 triples. Have pupils distinguish precisely: each displayed row is checked; the statement about all integers, an infinite set, remains a conjecture.

From 15 to 22, investigation alone then in pairs. Instruction: “Choose a letter for one of the three numbers; write down what it represents; write a complete proof ending with a ‘for every integer’ conclusion.” While circulating, code R1–Rk and prioritise spotting one proof naming the first integer and one proof naming the middle one.

Anticipating functions.

P1. calculated cases and the conjecture “three times the middle number”;

P2. possible unwarranted conclusion from the displayed cases alone; if it does not appear, T puts the statement “these 200 cases are enough” up for discussion, without attributing it to a pupil;

P3. proof from the first integer: $n, n + 1, n + 2$, then $3(n + 1)$;

P4. proof from the middle integer: $m - 1, m, m + 1$, then $3m$.

Phase 4 (22–36). Neriage: comparing two complete proofs

Select actually available answers and keep them side by side. If one representation is missing, ask the class: “what if the letter stood for a different number in the triple?” Any construction subsequently carried out by the class is marked T.

Q1. (P1) “What statement do the calculations make plausible? What do they actually prove?”, Expected: they prove each of the three cases; they suggest the universal conjecture without proving it.

Q2. (P2 or T) “Which integers do the 200 rows cover? Does the set of integers have a last row?”, Expected: the target domain is infinite; a finite non-exhaustive list does not cover “for every integer”.

Q3. (P3) “What does n represent? Why is $n + 1$ an integer? Where does divisibility appear?”, Expected: $n + (n + 1) + (n + 2) = 3n + 3 = 3(n + 1)$; $n + 1$ is an integer, so the result is 3 times an integer. If a pupil concludes right at $3n + 3$ by stating that $3n$ and 3 are two multiples of 3 and that their sum is still a multiple of 3, their proof of divisibility by 3 is complete and must be kept: factorising is not the only valid justification.

Q4. (P4) “What does m represent? How is it linked to n ? Which expression shows three times the middle number most directly?”, Expected: $(m - 1) + m + (m + 1) = 3m$ and $m = n + 1$; the two proofs are the same structure seen from two different numbers.

IDEAL BOARD (final state): to project or reproduce

LEFT: data and scope

Kadai: The sum of three consecutive integers always seems to be three times the middle number. How can we prove this for all integers without being able to test them all?

$$(-8) + (-7) + (-6) = -21 = 3 \times (-7)$$

$$7 + 8 + 9 = 24 = 3 \times 8$$

$$199 + 200 + 201 = 600 = 3 \times 200$$

Spreadsheet: B2=A2+1, C2=A2+2,
D2=A2+B2+C2.

200 cases are checked; the statement for all integers remains to be proved.

CENTRE: two linked proofs

R? or T: “the 200 cases are enough”: no, they do not cover the infinite set of integers.

P3, first integer n :

$$n + (n + 1) + (n + 2) = 3n + 3 = 3(n + 1).$$

P4, middle integer m :

$$(m - 1) + m + (m + 1) = 3m.$$

In both proofs: $m = n + 1$.

The quotient by 3 is the middle number.

RIGHT: matome and variation

Matome:

1. checked cases $\rightarrow$ conjecture;
2. arbitrary integer and role of the letter;
3. justified chain and recognised form;
4. “for every integer” conclusion.

Four consecutive integers:

$1 + 2 + 3 + 4 = 10$ refutes “always a multiple of 4”.

$4n + 6 = 4(n + 1) + 2$ specifies the remainder.

Phase 5 (36–43): Protected matome and deferred copying

Have pupils formulate the four steps before giving the written summary. The Kadai, the scope of the spreadsheet, the two choices of letter and the link $m = n + 1$ remain visible throughout the copying.

TO COPY INTO THE NOTEBOOK (word for word)

In a statement about all integers, finitely many cases only verify those cases and make a conjecture plausible. To prove it here: 1) we choose an arbitrary integer and state what the letter represents; 2) we write the numbers in terms of the letter; 3) we chain justified equalities and recognise the resulting form; 4) we conclude “for every integer”.

Thus, if n is the first integer, $n + (n + 1) + (n + 2) = 3(n + 1)$; if m is the middle one, $(m - 1) + m + (m + 1) = 3m$. In both cases, the sum is three times the middle number. A single counterexample is enough to refute a universal claim.

Phase 6 (43–51). Variation: four consecutive integers

Have pupils work individually on 5.4 f): “Is the sum of four consecutive integers always a multiple of 4?” A single case, $1 + 2 + 3 + 4 = 10$, is enough to refute the universal claim. Nevertheless, continue the algebraic calculation to obtain the stronger information:

$$n + (n + 1) + (n + 2) + (n + 3) = 4n + 6 = 4(n + 1) + 2.$$

The sum therefore always leaves a remainder of 2 when divided by 4; it is always even. Distinguish the role of the counterexample, sufficient to refute, from that of the formula, which characterises all cases.

Phase 7 (51–55). Exit: transferring the middle-number choice

On a half-sheet: “Let m be an integer. Prove in one line that the sum of the five consecutive integers centred at m is a multiple of 5.” Expected:

$$(m - 2) + (m - 1) + m + (m + 1) + (m + 2) = 5m.$$

Require the closing sentence “therefore, for every integer m ...” and the meaning of m .

Differentiation. Struggling pupils: grid “what the letter stands for / three numbers / calculation / recognised form / conclusion”, with no algebraic line provided. Fast pupils: give two proofs of the four-integer variation, then explain what each establishes beyond the single counterexample. Track H, optional: try the “Modify, create, solve” block following 5.1; do not open 5.5.

Homework, capped workload. The family block carries the full R prescription: 5.1 a)–d), worked solutions closed, then 5.2. Track M: reread the written summary and the two proofs of 5.4; do the S8 evening drills; do only 5.1 a)–d), worked solutions closed, then stop before 5.2. Track H, optional: try the “Modify, create, solve” block following 5.1. No Brevet problem is added tonight; 5.5 stays closed until the S9 Kadai, and reading the course notes pp. 25–26 (models J1 and J2), which would give its method, waits until the evening of S9. The full worked solutions for the exercise sheet remain closed and will be handed out after reviewing Arthur in S9.

Read: today’s written summary and the two proofs of 5.4.

Evening drills: “S8 evening” block, upside-down answers to check after trying.

Do: 5.1 a) to d), worked solutions closed, then 5.2.

Optional: the “Modify, create, solve” block following 5.1.

The full worked solutions for the exercise sheet will be handed out after reviewing 5.1; no Brevet problem is added.

If running late. First cut the 5.3 revisit to three minutes, then the variation to five minutes. Investigation, comparison of the two choices of letter, matome and exit remain incompressible. Arthur, problem posing and handing out the worked solutions are never added to this slot.

Success criteria... on the exit slip, the pupil names the five integers, obtains $5m$ and concludes for every integer; can explain what the 200 rows actually verified and link the two central proofs via $m = n + 1$.

Teaching note: [S] The same problem is set for everyone, but choosing the first integer or the middle one allows constructions of different sophistication that can be compared publicly: the teacher accepts and values a “wide range of degrees of sophistication”, open tasks enabling everyone to formulate their own strategy. [T] The neriage selects, orders and connects actually available answers; it does not turn a chain of phases into P1–P4. [M] The data, the two proofs and the relationship between variables remain simultaneously visible so that the board makes the architecture of the argument legible. [L] The statement about examples is restricted to a universal statement over the infinite set of integers: the tested cases are well established, but they do not cover the entire domain.

Out-of-slot reserve. The “curious subtraction” $10a + b - (10b + a) = 9(a - b)$ and calculation-program problem posing are two possible problem-based lessons, each with its own Kadai, investigation, neriage, matome and Bansho. They are not extra activities to squeeze in after 5.4.

S9: Returning to Arthur, comparing proofs and Brevet 5 (format B, 55 min)

Pupil objective. Prove a divisibility result; compare two general proofs without confusing validity, length and readability; write up Brevet 5 under exam conditions and then assess one's own written work against explicit criteria.

Skills: investigating, reasoning, communicating.

Materials. Two real, anonymised pieces of pupil work on 5.1, chosen after a quick collection; worked solutions to the exercise sheet kept in an envelope until the end of the review; statement of 5.5 projected while the exercise sheet remains closed; Brevet 5; groups of 3; timer.

Phase 1 (0–10). Returning to Arthur: trials, proof and conclusion

Quickly collect or photograph 5.1, then compare two pieces of work actually available: ideally one answer based only on $x = 2$ and $x = 3$, and one algebraic chain. If one is missing, the teacher writes up a proposal T without fabricating pupil work. Have the complete proof corrected, and only then hand out the worked solutions to the exercise sheet.

Q1. “What exactly do the two trials prove? Which line deals with any number? Where do we read the universal conclusion?”, Aim: immediately reinvest the S8 matome. Expected: the trials establish two cases; if x denotes any number, expanding and then simplifying the expression from the calculation program and Arthur's expression makes it possible to compare them for any value of x ; the conclusion must state this domain.

Phase 2 (10–13). Handing over 5.5: from trials to the proof question

Keep the exercise sheet closed: its hint announces the method and would remove the choice to be observed. Project only Anabelle's statement and the three pairs. Have pupils calculate 16, 32 and 48, then write the Kadai: “Anabelle claims that the difference of the squares of two consecutive odd numbers is always a multiple of 8. How can we prove it, and what does each method make visible?” Have pupils state the scope: the three cases are established; the general statement remains to be proved.

Phase 3 (13–21). Investigation: alone, then in threes

Four minutes alone, then four minutes in threes. Ask for a written proof with a defined letter, a chain of equalities and a conclusion. While circulating, code the answers R1–Rk without names or initials; spot expansion, factoring out the result and difference of two squares. Give no hint before having visited all groups. If no general proof appears after six minutes, have everyone publicly open the hint for 5.5, ask which identity deals specifically with a difference of two squares, and then label T any construction resulting from this prompt.

Phase 4 (21–34). Neriage: validity, economy and information made visible

Select and order the pieces of work actually available. P1–P4 denote functions in the discussion; they do not presuppose four authors or four scripts.

P1. The trials 16, 32, 48 and the conjecture; they remain on the left as given data.

P2. Expanding gives $8n + 8$. This form already allows a complete proof if the pupil writes: $8n$ and 8 are two multiples of 8; their sum is therefore a multiple of 8.

P3. $8n + 8 = 8(n + 1)$, with $n + 1$ an integer: the quotient is explicit. This line improves readability; it is not the only valid justification.

P4. $[(2n + 3) + (2n + 1)] \times [(2n + 3) - (2n + 1)] = (4n + 4) \times 2 = 8(n + 1)$: same result, but the sum and the gap between the two odd numbers are visible.

Q2. “Does $8n + 8$ already allow us to conclude, even if the 8 is not yet written in front of a bracket?”, Aim: not to disqualify a correct proof. Expected: yes, provided one invokes the closure of multiples of 8 under addition; otherwise the chain remains without a conclusion. Factoring out provides a second, more compact conclusion.

Q3. “What can we compare between two proofs that are both valid?”, Expected: their number of steps, the knowledge used and the information they make visible. Direct factorisation shows “sum $\times$ gap”; expansion is more automatic.

Phase 5 (34–39): Protected matome and copying

Have pupils formulate and then copy: trials support a conjecture; a general proof deals with any integer; divisibility can be established through the form $8k$ with k an integer or through a previously justified property of multiples; a shorter or more illuminating method does not make the other one wrong.

Phase 6 (39–49). Brevet 5 under exam conditions: ten genuinely silent minutes

Have pupils write up the three questions, alone, without a calculator, with a visible timer. The worked solutions handed out after Arthur remain closed and no answer appears on the board before the signal. This phase assesses transfer: two consecutive odd numbers are still $2n + 1$ and $2n + 3$, but this time pupils must study their sum and deal with a false statement using a counterexample.

Phase 7 (49–55): Review, self-correction and grid

Only after the signal, have pupils open the worked solutions. Write on the board the central form $4(n + 1)$ and the counterexample $1 + 5 = 6$. Each pupil compares their written work, circles one criterion achieved and underlines one to rework; collect a sample of scripts to check what the grid actually makes visible.

Quick answers (teacher): a) $7 + 9 = 16 = 4 \times 4$, $15 + 17 = 32 = 4 \times 8$ b) Let n be an integer; $(2n + 1) + (2n + 3) = 4(n + 1)$ and $n + 1$ is an integer, so the sum is a multiple of 4 for any integer n c) no: $1 + 5 = 6$ is not a multiple of 4. Going further, $(2n + 1) + (2p + 1) = 2(n + p + 1)$ only shows that the sum of any two odd numbers is even.

Q4. “Is checking on an example part of the proof of b)?”, Expected: no. It can serve as a check after the reasoning, but the general proof relies on the defined letter, the equalities, the recognised form and the conclusion. In c), the counterexample has a different status: it is enough to refute the universal statement.

IDEAL BOARD (final state): to project or reproduce

LEFT: return to Arthur, data and Kadai

Review 5.1:

$$5(-2x + 5) = -10x + 25;$$

$$(x - 5)^2 - x^2 = -10x + 25.$$

The trials at 2 and 3 establish two cases; the identities prove the equality for any real number x .

Kadai: Anabelle claims that the difference of the squares of two consecutive odd numbers is always a multiple of 8. How can we prove it, and what does each method make visible?

Let n be an integer: two consecutive odd numbers are written $2n + 1$ and $2n + 3$.

$$5^2 - 3^2 = 16, 9^2 - 7^2 = 32, 13^2 - 11^2 = 48: \text{conjecture.}$$

CENTRE: compared proofs on 5.5

P2, expand:

$$(2n + 3)^2 - (2n + 1)^2 = 8n + 8.$$

$8n$ and 8 are multiples of 8; so is their sum.

P3, explicit quotient:

$$8n + 8 = 8(n + 1), \text{ with } n + 1 \text{ an integer.}$$

P4, factorise first:

$$\begin{aligned} &[(2n + 3) + (2n + 1)] \\ &\quad \times [(2n + 3) - (2n + 1)] \\ &= (4n + 4) \times 2 = 8(n + 1). \end{aligned}$$

Same result; sum and gap visible.

RIGHT: matome and Brevet grid

Matome:

1. trials support a conjecture, not the general proof;
2. divisibility can be shown via a sum of multiples or via $8 \times k$ with k an integer;
3. one method is shorter or more illuminating without making the other wrong.

Brevet grid:

- letter and domain;
 - algebraic translation;
 - justified equalities;
 - recognised form and integer obtained;
 - quantified conclusion.
- Numerical check: useful after the proof, never required to prove it.

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Trials can support a conjecture; a general proof chooses any integer and chains together equalities that hold whatever its value.

To show that an integer is divisible by 8, we can write it as $8 \times k$ with k an integer; we can also establish that it is a sum of multiples of 8. Factorising using $a^2 - b^2$ is shorter here and shows the sum and the gap, without making expansion wrong.

Brevet grid: letter and domain • algebraic translation • justified equalities • recognised form and integer obtained • quantified conclusion. A numerical check is optional after the proof; it does not replace it.

Differentiation. Struggling pupils: after individual investigation, have them complete orally “let n be an integer / the two odd numbers are / I calculate / therefore” without giving the central chain. If no route gets started after six minutes, open the hint for 5.5 for everyone. Fast pupils: write a second proof, then explain precisely what factorisation shows in addition. 5.6 remains an H reserve outside the lesson, not a second investigation to squeeze in.

Homework, capped workload and announced review. For everyone: read the course notes pp. 25–26 (models J1 and J2, example of consecutive even integers), reread 5.5 and annotate one’s script for Brevet 5 using the grid; do the S9 evening drills; write up Brevet 6, worked solutions closed, which will be reviewed at the start of S10. Track M: same work, stopping at the 35–40 minute threshold, and written attempts never penalised. Track H, optional and by choice: try 5.6, or write up Brevet 2 (calculation program, shorter): in both cases, compare

only after writing with the worked solutions now handed out. No cross-curricular task or second challenge is added tonight.

Read: course notes pp. 25 to 26, writing a proof and model J2 on consecutive even integers; then the proof of 5.5 and your work on Brevet 5.

Evening drills: “S9 evening” block, upside-down answers to be checked after trying the questions.

Do: all of Brevet 6, worked solutions closed.

Rework: circle in the grid one criterion achieved and underline the one that still needs reworking.

Optional: 5.6, “Divisible by 7”.

Optional, as an alternative to the previous one: Brevet 2, “Calculation program”, shorter.

Brevet 6 will be reviewed at the start of the next lesson.

If running late. Preserve Arthur, the investigation, at least one general proof discussed, the matome and the ten minutes for the Brevet task. Reduce the neriage to two routes actually available and the final self-correction to four minutes; never reintroduce 5.6 into the slot.

Success if... the class distinguishes what the three trials prove from what a general chain proves; an answer ending with $8n + 8$ is judged on its logical conclusion and not rejected mechanically; the Brevet 5 sample makes it possible to identify, for each script, the first missing criterion.

Teaching note: [S] Pupils judge a lesson “good” when the teacher completes the D–S–R cycle: whole-class exposition, individual work or homework, then review and feedback, specifically including the use of pupils’ work or group presentations to give feedback to an individual pupil or to the whole class. Among the six functions of homework recognised by pupils is “self-assessing and learning from one’s mistakes”. Here, Arthur is genuinely used before the new investigation and the Brevet task receives immediate feedback. [M] A single problem carries the comparison: selecting, ordering and connecting answers must not become a succession of unrelated solutions. [L] The board keeps the tools, the proofs and their links so as to make the organisation of the reasoning visible. [K] Give the pupil time to discover the correct solution for themselves rather than pointing out the mistake, and do not rephrase a good idea in their place.

S10: Review of Brevet 6, choice of form and bridging question (format B, 55 min)

Pupil objective. Compare equivalent expressions without inventing an absolute “best form”; choose an expression according to the value and the aim; distinguish an identity from an equation; separate, in $x^2 - 9 = 0$, verified solutions from exhaustiveness that still needs justifying; build a targeted revision plan. **Skills:** calculating, reasoning, communicating, self-assessing.

Materials. Scripts for Brevet 6 with the initial written attempt kept, even if the worked solutions were consulted afterwards; seating plan to code R1–Rk; set 4.1; exercise sheet 9.1; course notes part 1; **Summary sheet handed out today**; multiple-choice quiz; timer; exit ticket “my priority rework”.

Phase 1 (0–15). Real review of Brevet 6: three forms, no declared winner

Write the Kadai: “For $I = (7x - 3)^2 - 25$, is there a form that is always better, or does the choice depend on the value and the aim?” It remains visible until the matome. During the first three minutes, have pupils reopen Brevet 6 and circle the work written before consulting the worked solutions; any subsequent annotation remains visible in another colour. Quickly collect or photograph several answers and code R1–Rk without names or initials. Select functions actually available; if one is missing, ask a public question signed T, without fabricating pupil work.

P1. The expanded form $49x^2 - 42x - 16$, obtained through a complete chain;

P2. The factorised form $(7x + 2)(7x - 8)$, with the block $(7x - 3)$ recognised;

P3. For $x = 1$, reasoned comparison of the starting, expanded and factorised forms;

P4. For $x = \frac{3}{7}$, starting form $0^2 - 25$ and factorised form $5 \times (-5)$.

Q1. “What establishes that the three expressions represent the same number for any x ?”, Expected: the chains derived from $(a - b)^2$ and $a^2 - b^2$; agreement for a single value does not prove the identity.

Q2. “For $x = 1$, can we award the title of ‘fastest’ form without a criterion?”, Expected: no. The factorised form makes -1 visible; the starting form uses $4^2 - 25$; the expanded form gives two subtractions. A preference must state what it counts or what it wants to make visible.

Q3. “What do the two calculations for $x = \frac{3}{7}$ show?”, Expected: the starting form reveals the inner zero; the factorised form reveals two opposite factors. Both are direct. There is no best form independently of the value and the aim.

Phase 2 (15–19): Matome for Brevet 6 and script annotation

Have pupils formulate and then copy: equivalent expressions preserve the value of the expression; we choose a form according to the required calculation and the information it makes visible (zero, factor 1 or -1 , simple numbers, structure to be proved). A form that is more convenient in one case is not “truer” and is not necessarily better in another. Then open the worked solutions: each pupil corrects a single priority line without erasing their initial work.

Phase 3 (19–26): Redoing 4.1 in fixed time, without new fluency items

Redo the same set 4.1 as in S5 for five minutes exactly. Then project only the answers for two minutes of personal correction. Record $n(\geq 6/8)$ and $n(8/8)$, then compare with the S5 indicators: the measurement focuses on accuracy in a fixed time, not on a speed ranking. Do not add the three fluency questions from the old version: they would take away time from the review actually announced.

Phase 4 (26–35). 9.1: analysing a justification, not re-enacting a discovery

Since the complete worked solutions have been available since S9, acknowledge this fact and allow pupils to consult them. The task is not to secretly rediscover the results, but to check their scope. Have pupils work for three minutes alone and then six minutes in pairs on the question: “In your initial work and in the worked solutions, which sentences establish that 3 and -3 are solutions? Which sentence justifies the factorised identity? Where is a justification still missing if we want to claim that all solutions have been found?” Ask for three columns on the sheet: verified / conjectured / proved; each classification must refer to a specific equality or sentence.

Q4. “What do the substitutions $x = 3$ and $x = -3$ prove? What does $(x + 3)(x - 3) = 0$ make visible?”, Expected: the two values are indeed solutions; the factorisation makes visible the two factors that may equal zero. On its own and without an additional property, it does not provide the exhaustiveness argument.

Phase 5 (35–46): Comparing statuses, then institutionalising identity and equation

Select the formulations actually available and place them in three columns. Under verified: $3^2 - 9 = 0$ and $(-3)^2 - 9 = 0$. Under proved: the identity $x^2 - 9 = (x + 3)(x - 3)$, valid for any real number x . Under conjectured: “if a product is zero, at least one factor is zero”. Keep this last sentence in a dotted box “to be proved in Chapter 02”; do not erase it, but do not state it as a theorem and do not use the notation $\mathcal{S}$.

Q5. “Can we say today that 3 and -3 are solutions? Can we say that factorisation alone proves that they are the only ones?”, Expected: yes for the first sentence; no for the second as long as the zero-product implication has not been established. If another rigorous proof of exhaustiveness appears, keep it and check each step: the future property is not the only possible proof, only the intended general tool.

Q6. “What is the status of the letter in $(a + b)^2 = a^2 + 2ab + b^2$ and in $x^2 - 9 = 0$ ”, Expected: an identity states equality for all values in a domain; an equation imposes a condition on an unknown, and solving consists in finding all values in the domain that satisfy it. Depending on the equation, the solution set may be empty, contain one or more values (possibly infinitely many) or coincide with the entire domain. Have pupils copy the matome. Guided reading of the course notes on the four uses of the $=$ sign then becomes precise knowledge, not the false slogan “an equation is true for certain values only”.

H reserve outside the slot. Brevet 9 and 9.2 remain in the bank. They replace neither the review of Brevet 6 nor the bridging question. A pupil who tries them after their essential revision must distinguish, there too, verified values, factorisation identity and proof of exhaustiveness.

Phase 6 (46–55): Announcing the test and choosing a priority rework

Hand out the Summary sheet, present the five required families, then leave two silent minutes: each pupil chooses a single rework route A, B or C and writes down the exact item they will redo. Collect the tickets before pupils leave; “revise everything” is not an acceptable plan.

IDEAL BOARD (final state): to project or reproduce

LEFT. Brevet 6: three correct forms

Kadai: For $I = (7x - 3)^2 - 25$, is there a form that is always better, or does the choice depend on the value and the aim?

P1, expand:

$$I = 49x^2 - 42x - 16.$$

P2, factorise:

$$I = (7x + 2)(7x - 8).$$

Both equalities are established by the identities; numerical agreement alone does not prove them.

CENTRE: choice of form and bridging question

P3, $x = 1$:

$$\text{starting} = 4^2 - 25 = -9;$$

$$\text{expanded} = 49 - 42 - 16 = -9;$$

$$\text{factorised} = 9 \times (-1) = -9.$$

Preference to be justified, with no imposed winner.

P4, $x = \frac{3}{7}$:

$$\text{starting} = 0^2 - 25 = -25;$$

$$\text{factorised} = 5 \times (-5) = -25.$$

Bridging question 9.1:

Verified: 3 and -3 satisfy $x^2 - 9 = 0$;

Proved: $x^2 - 9 = (x + 3)(x - 3)$ for any real number x ;

To be justified: the absence of any other solution by this route alone.

RIGHT: statuses and targeted revision

Matome.

Identity: equality stated for all values in a domain.

Equation: condition on an unknown; solving means finding all its solutions.

Conjecture to be proved in Ch. 02: if a product is zero, at least one factor is zero.

Revision:

Active summary sheet + S10 evening drills + quiz, then a single rework: A calculation / B proof / C choice of form.

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Equivalent expressions have the same value, but they do not make the same information visible. We choose a form according to the value, the aim and a stated criterion; no form is always the best.

An identity is an equality stated for all values in a domain. An equation is a condition on an unknown; solving it consists in finding all values in the domain that satisfy it.

For $x^2 - 9 = 0$, 3 and -3 are verified solutions. The identity $x^2 - 9 = (x + 3)(x - 3)$ is proved. Exhaustiveness via the product form will use in Chapter 02 a property that is here only an explicitly flagged conjecture.

Homework, 35–40 minutes at most, without accumulation. For everyone: 1. Summary sheet using active recall, eight to ten minutes; 2. S10 evening drills then self-marked multiple-choice quiz, twelve to fifteen minutes; 3. A SINGLE rework chosen on the exit ticket, twelve to fifteen minutes. Route A (calculation): redo only the missed items from 4.1 and two corresponding quick questions; if no items were missed, Brevet 1 (brevet-style quick questions) or Brevet 4 (justified clever calculation), which have not yet been set. Route B (proof): rewrite

question 5.4 d) using the grid, then compare with the model. Route C (forms): correct Brevet 6 c)–d) and explain in two sentences what each form makes visible. Track M: model allowed and stop at the 35–40 minute threshold, written attempt never penalised. Track H, optional and with no obligation: offer Brevet 8, “The product of the two neighbours”, a direct transfer of proofs on integers; tasks 7 and 9 remain in the bank, announced but not due.

Read: the Summary sheet, then the multiple-choice quiz in the course notes p. 27.

Learn by heart: the three identities and the standard write-up of a proof.

Evening drills: “S10 evening” block, upside-down answers to be checked after trying the questions.

Rework: a single route from the exit ticket, calculation with the missed questions from 4.1, or failing that with Brevet 1 or Brevet 4; proof with 5.4 d); choice of expression with Brevet 6 c) and d).

Optional: Brevet 8, “The product of the two neighbours”.

Test in the next lesson, without a calculator, on the whole chapter.

If running late. First drop the whole-class comparison of the two 4.1 indicators, then reduce its correction to displaying the answers. Never drop the review of Brevet 6, its matome, the distinction “verified solutions / exhaustiveness”, the precise definition of identity and equation, or the personal choice of a rework before the test.

Success if... a preference for a form is accompanied by a criterion; the class can say that 3 and -3 are verified without attributing unproven exhaustiveness to factorisation alone; each ticket names a route A, B or C and a specific item; no pupil has been given Brevet 9 as a common requirement.

Teaching note: [R] The D–S–R cycle (exposition, work, review and feedback) and the use of pupils’ work for feedback require making use of the announced Brevet 6, not abandoning it in favour of a new task: among the teacher’s actions is explicitly included the use of pupils’ work or group presentations to give feedback to an individual pupil or to the whole class. [N] What is established is assessment DURING instruction, focusing on the process rather than the result alone, through observation of discussions and written work. Self-assessment, for its part, features among the alternative strategies actually trialled. [B] The answers actually available are selected, ordered and connected; a missing route becomes T, never fabricated pupil work. [M] The three forms, their criteria and the proof boundary remain simultaneously visible. Brevet 9 therefore remains H outside the slot; 4.1 measures accuracy in a fixed time, and final revision is targeted rather than cumulative.

S11: Test (50 min in a 55-min slot)

Administration. Version V3 paper in force (5 exercises, 20 marks, mark scheme displayed: verified total = 20, no dependency between exercises). Calculator not allowed. Exact breakdown of the slot: 0–3 settling in and reading instructions; 3–53 individual sitting, i.e. the announced 50 minutes; 53–55 collection. Single starting instruction: “read the whole paper before starting; in the proof, define your letter precisely before calculating”.

During the test. Note which questions cause widespread blocks (material for the review): watch Ex. 3 e (counter-example to $(3a + b)^2 = 9a^2 + b^2$, Amir’s claim) and Ex. 4 c (the proof): these are the two questions that diagnose the chapter. Note who finishes very early (candidates for the part 8 challenges later on).

Marking the scripts (analysis grid). Code each error encountered: **N** nature / opposite / sign; **D** expansion (distributive property, identity, middle term, square of a product); **F** factorisation (incorrect product, misrecognised identity, explicitly requested factor not brought out); **P** proof (undefined letter, algebraic translation, quantifier, examples mistaken for a proof); **C** clever calculation (identity not used or unjustified calculation). A correct factorisation is never marked wrong on the sole ground that a larger factor could be extracted. Counting by code serves to choose the rework; it does not replace reading the reasoning.

Material preparation for the feedback session. After marking, choose ONE written attempt actually found in the scripts, rich enough for a local repair to be possible. Copy it faithfully onto the board, removing any name, initial, script number and identifiable detail. If the first attempt considered does not allow a fruitful discussion, choose another from the scripts: never fabricate pupil work. Before pupils enter, place the scripts face down; they are only opened in the ninth minute.

End of week 3: alternating exam-paper weekend (Brevet 3). This is the first weekend where choosing a form (model V1, question c) of the paper) has been seen in class, in S10, and where the worked solutions to the exercise sheet, handed out in S9, are open for self-correction. Brevet 3 under exam conditions (worked solutions closed while writing), followed by ten minutes of correction using the worked solutions and the error log, replaces all other work; it is not added on top. In the default case, the test ends the week: this weekend therefore follows the test and precedes the return of scripts; the paper serves as a review of the chapter and a stitch towards Chapter 02 (question c) reads a zero factor), with no work from the next chapter. If the timetable places this weekend between S10 and S11, Brevet 3 replaces the A/B/C rework from the evening of S10, as it covers all three routes. If the timetable exceptionally prevents this paper, rest.

Weekend paper variant (to be pasted in place of the “Evening drills” and “Do” lines).

Do: Brevet 3 in one sitting, worked solutions closed.

Rework: your script using the worked solutions to the exercise sheet, then one error in the error log.

This paper replaces all other weekend work; it is not added to it.

Returning scripts: first 10 minutes of the next lesson. 0–2: two pupils rephrase exactly what is written, without correcting yet (P1). 2–5: gather the locations or diagnoses actually suggested, keeping at most two for comparison; have pupils identify the first unjustified equality or sentence and the corresponding code (P2). 5–8: build the minimal repair, then choose a suitable check: re-expanding a factorisation, substituting a value to look for a discrepancy, or checking the scope of a conclusion (P3). 8–9: formulate a prevention rule and test it on a neighbouring example built from the real error (P4). 9–10: pupils open their scripts, circle ONE personal priority and note its code. Announce the two or three dominant codes. A single entry in the error log, corresponding to this priority, is required for the next lesson; it will be checked. **Stitch to Ch. 02:** later in the year, fluency work on the zero-product property will notably revisit $(x - 7)(x + 4) = 0$, whose general property was still only a conjecture in S10.

Kadai for returning scripts (absent during the test). For the delayed feedback: which real error should we choose, where does its first breakdown occur, and what rule will prevent it from happening again?

IDEAL BOARD (final state): to project or reproduce

LEFT: written work and diagnoses

Kadai: For the delayed feedback: which real error should we choose, where does its first breakdown occur, and what rule will prevent it from happening again?

[real written work, faithfully copied and completely anonymised]

P1: “What exactly does this written work say?”

P2: “Where is the first unjustified equality or sentence?”

Dominant codes from the test:

N ___ D ___ F ___ P ___ C ___

CENTRE: repair and check

P3: “What is the smallest change that makes the line correct?”

“Which check is suitable here?”

o re-expand; o substitute;

o check the scope of the conclusion.

Write down the check actually chosen and its result.

RIGHT: prevention and priority

P4: “In what neighbouring problem could this error come back?”

Prevention _____ rule: _____

When opening the script:

my priority code: _____

the circled question: _____

For the next lesson: a single complete entry in the error log.

Reactivation starter for the following week. The content depends on the scripts. If **P** is among the dominant codes, reuse the structure of 5.4 with a modified property and data. Otherwise, choose a single short item falling under the actual dominant code N, D, F or C. This starter replaces the usual opening fluency routine; it is not added to it. It is formative by default. Any grading must be announced in advance along with its criteria and an opportunity to rework.

Evening drills: none on the evening of the test.

Do: no written exercise before the script is returned.

Rework: after the script is returned, a single priority question in the error log with the problem statement, the answer given, the correction and the cause of the error.

The next lesson will begin with the study of a real, anonymised error.

Teaching note: [R] Rather than pointing out the error, give the pupil time to discover the correct solution for themselves, and assess during instruction by observing the process rather than the result alone. [C] The D–S–R cycle, the use of pupils’ work for feedback and “learning from one’s mistakes” as a recognised function of homework all point the same way. [B] Pieces of work are selected, ordered and connected based on what pupils actually produced; a missing route remains a teacher intervention, never fabricated pupil work. [M] The spatial separation between written work, diagnoses, repair and rule makes it possible to keep the problem while making the progression of the reasoning legible. Hence a short clinic on a real script, followed by a single achievable personal priority.

5. Permanent routines (for display)

1. Timed opening fluency drill: marked immediately when it is practice (2.1, 3.1, **4.1**). The S0 diagnostic is collected first; the first sorting task 1.1 is followed by a *neriage*.
2. “**The photographer**” before any factorisation: how many terms? Are the outer terms squares? Is the middle term the double product?
3. “**Which check, which conclusion?**”: I expand back to establish an identity; I can substitute a value to look for a discrepancy. Discrepancy $\Rightarrow$ false; one-off agreement $\nRightarrow$ proof.
4. “**Who did it differently?**” after any solution (expand or factorise? geometric or algebraic proof?).
5. “**What changes? What never changes?**” after any family (2.3, 3.3. . .).
6. A *numerical counterexample* is enough to disprove; no number of examples is enough to prove: repeated at every “false” and at every proof.
7. Never “do you understand?” $\rightarrow$ “**how do you know?**” / “why this identity and not another?”.
8. Stuck pupil: ask the SAME open question to another pupil, then have the first one rephrase (no funnel of micro-questions).
9. Homework is ALWAYS followed up on: never homework without feedback.
10. The board tells the story of the lesson: nothing is erased before the summary (only the rough-work area), and pupils copy at the end.
11. **Homework monitoring** (carried over from Ch. 00, nothing to set up again): at most one retrieval flash quiz per week (5 min, a homework exercise used as a reference structure, with figures possibly changed, formative by default; any grading is announced in advance according to the objective, with criteria and an opportunity to redo, specified in advance on the end-of-week sheet) + *3–5 exercise books checked in each lesson* on rotation, looking at evidence of

working, never neatness.

12. **Displayed kadai (kit K1)**: in each capstone lesson, write on the board (left area, boxed) the generic question of the day, which the matome must answer. See the ideal boards S1, S3, S5, S8.
13. **Co-constructed matome (kit K2, 3 min in capstone lessons)**: before copying the written summary, (1) reread the kadai; (2) two pupils suggest an answer sentence, the class refines it, the teacher writes it in the right-hand area using the pupils' words; (3) compare with the handout ("what makes the handout's definition more precise than ours?"), then copy the official written summary unchanged. The final written summary does not change; only how it is generated changes.
14. **Show-of-hands survey (kit K4)**: after each method presented, "who used this method? raise your hand": validates those who did not present, gives a snapshot of the class, and legitimises the follow-up "is there a faster, safer way?".

6. Remediation script: the 6 major errors of the chapter

Ready-to-use focusing prompts (never a direct correction: a question that refers the pupil back to a check they can do on their own).

The pupil writes / says...	The teacher replies...
$(a + b)^2 = a^2 + b^2$ (the classic error)	"Take $a = 3$, $b = 4$: calculate $(3 + 4)^2$, then $3^2 + 4^2$. Same numbers? Then how much is missing, and where is it on the drawing of the square?"
$(2x)^2 = 2x^2$	"Write $(2x)^2 = 2x \times 2x$. How many 2s? How many x s? The square applies to the whole factor."
$-(5 - 4x) = -5 - 4x$	"The opposite changes the sign of each term. Rewrite $-(5 - 4x)$ as $-1 \times (5 - 4x)$ and expand."
factorises $x^2 - 64$ as $(x - 8)^2$	"Expand $(x - 8)^2$ back: how many terms do you get? And how many does $x^2 - 64$ have? Two terms $\rightarrow a^2 - b^2$; three terms $\rightarrow$ square."
writes $4x^2 - 4x + 1 = 4x(x - 1)$	"Expand your answer back. Do you get the $+1$ back? Test $x = 1$ in both: same values?" (Louise's work)
"I checked on three examples, so it is true"	"Three examples test three cases. The statement is about all integers. Can the spreadsheet test them all? What is needed instead of a number?"

7. Required spoken vocabulary (across the lessons)

- term / factor
- sum / product (type)
- expand / factorise
- simplify (collect like terms)
- identity (true for all values)
- double product ($2ab$)
- perfect square
- opposite of an expression
- common factor
- conjecture / counterexample
- prove / "for all n "
- even / odd / consecutive

Rule: from the lesson in which a word is formalised, a pupil's answer that avoids it ("the little number in the middle") is rephrased by the pupil themselves using the exact word. The word identity, formalised in S3 and then reused in S4, is crucial: it is what distinguishes "for all a, b , $(a + b)^2 = a^2 + b^2$ " (false: a single counterexample suffices) from "there exist $a, b \dots$ " (2.4 g, true for $b = 0$).

8. Ongoing monitoring and formative assessment

- **Exit checks** (mini-whiteboard, S1/S3/S4/S7): the success rate determines the opening of the next lesson; below 70 %, the next fluency drill revisits the missed item; above that, we move on.
- **The S4 votes** on 2.4 b, f and g: compare the distributions and especially the nature of the justifications before and after discussion. If the distinction between square of a difference / difference of squares or "for all" / "there exists" remains shaky, the written summary is not enough: schedule another flash question in S5.
- **Course notes micro-tests** (self-tests, answers provided): those in Part 1 (three flash-question boxes) remain available as self-tests and are not added to the S0 homework; Part 2 becomes available after S1; Part 3 only after S4; Part 4 after S6. In S10, the multiple-choice quiz is common to all; a micro-test is only revisited if it matches personal route A, B or C, never as a second cumulative list. At the start of the next lesson, "who understood their mistake?": ungraded.
- **Fixed-time mental maths with identities** (4.1): record in S5 the number of pupils with at least 6 correct answers out of 8, then 8 out of 8; repeat the same set during the same five minutes in S10. The goal is visible progress in accuracy, not a speed ranking.
- **Participation grid**: aim for every pupil to have come to the board at least once by the end of the chapter (drill, area proof, vote or proof), ticking off as you go.

- **After the test:** the error codes (N/D/F/P/C, S11 sheet) give the exact content of the fluency drills for the first 3 lessons of the next chapter (direct intercept theorem), as spaced retrieval; the link with Ch. 02 is direct (its fluency skills reopen $(x - 7)(x + 4) = 0$).

9. Pacing and management: margins, priorities, contingencies

Non-negotiable (never cut). The investigation, the neriage, an area proof, the algebraic proof and the matome in S3 • in S4, P1, P3, the two P4 chains and the matome, then the logical contrast f)–g) • in S5, P1, P2 or T, P3, the P4 transfer and the “valid / efficient” matome • in S7, the shared investigation on the four cards, P1–P4, the matome and the refutation / proof distinction in 3.6 • in S8, the investigation, the two choices of letter linked by $m = n + 1$, the matome and the exit check • in S9, Arthur, 5.5, the matome and the ten minutes on Brevet 5 (the brevet, French national exam at the end of Year 10) • in S10, the feedback on Brevet 6, its matome on choosing the right form, the “checked / conjectured / proved” boundary, the definitions of identity and equation and the targeted revision plan.

Pacing of investigations. Investigation time is scaled to the size of the question, not to the lesson template: in S8, six minutes of data and formulation precede seven minutes of proof alone and then in pairs; an investigation of representations and proofs, such as S3, requires more; eight minutes alone and then in threes suit 5.5 in S9. Exercise 5.6 remains outside lesson time. Too long an investigation on a short question produces drift, not reflection. The time saved protects first the neriage and the matome, then targeted practice.

Adjustment variables (in order of sacrifice). 1.4 ($\rightarrow$ R track homework) • P2 and the 2.2 applications in S4 ($\rightarrow$ differentiated track, without assigning an unexplained concept) • 3.4 c–d ($\rightarrow$ fast track) • the 5.3 review in S8 reduced to three minutes, then the four-integer variation reduced to five minutes; the two choices of letter in 5.4 stay together • 4.4 a ($\rightarrow$ independent extension of an R exercise; b, c and d are led in class in S5: b and c are cards *A* and *B*, d is the transfer) • the entire H and cross-curricular bank (it is a bank, not an obligation). Reminder of the ambition rule: cut the drill, never the capstone.

If the chapter overruns (disrupted start of term, events...): merge S6 and S7 while keeping the photographer, the central family of S6, the four sorted cards of S7 and Louise; 3.4 remains practice limited to homework. Never sacrifice S3 or S8.

If a class is ahead : open the “Going further” bank (Part 8 of the exercise sheet: 8.1 $\rightarrow$ 8.6) for independent work at the back of the classroom from S5 onwards; 8.1, 8.2, 8.4 and 8.6 are self-contained; 8.3 (conjugate expression) is a reward for presenting at the board. Exercise 4.6 (“fixed perimeter, maximum area”) complements the same bank on the clever-calculation side.

Struggling pupils identified in S0 (items 1, 2, 3, 8): full M track + 1.1 + 2.4 (both “conceptual” exercises are accessible to them: they do not require technical virtuosity); see them during support slots using the Reference sheet as the sole resource; call on them first for drills (not for proofs).

Time to copy the written summary: measurement, not trade-off. The matome phase contains two things: having pupils FORMULATE the summary, then having them COPY it. The written summaries in this guide have been measured character by character and converted at **70 characters per minute**, a cautious pace for neat Year 10 handwriting. The table below compares these durations with the minutes allocated by the sheet (identical to the timeline of the Bansho sheet for the same lesson). It does NOT authorise shortening the written summary: it indicates where the teacher must either dictate, provide a printed slip, or free up time earlier in the lesson.

Lesson	S0	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
Characters in summary	491	570	561	457	354	500	457	554	597	618	650
Estimated copying (min)	7	8	8	6,5	5	7	6,5	8	8,5	9	9
Allocated phase (min)	6	6	8	9	8	6	5	4	7	5	4 + 11

Three lessons fit comfortably (S3, S4, and S2 just barely). *S7 and S9 are the two pinch points*: copying there takes about twice the planned time, because the phase also serves another purpose (Louise in S7, Brevet 5 in S9). *S1, S5, S6, S8 and S0* are short by one to two minutes. Three possible solutions, to be decided by the teacher and not here: (1) dictate the written summary while the board remains displayed; (2) provide printed slips for the longest written summaries (S7, S9) and have pupils copy only the wording produced by the class; (3) carry copying over to the start of the next lesson, which the rule “nothing is erased before the photograph” makes possible since the board is photographed. The deficit is documented here, not resolved: the phase timings are the shared backbone of the guide and the twelve Bansho sheets, and are not modified.

Pupils absent for a test or a key lesson. The Reference sheet + micro-tests + multiple-choice quiz cover independent catch-up; for a missed S3, have a peer reconstruct the chain “data $\rightarrow$ conjecture $\rightarrow$ dissection $\rightarrow$ algebraic proof”, then redo the exit check; for S8, have the pupil redo 5.4 with the model solution in front of them.

10. What this guide assumes is in place

Current version of the course notes, exercise sheet and worked solutions (“expanding square” activity, vote 2.4, traps 3.3, proof 5.4, bridge question 9.1 included) • roadmap in the teacher version of the exercise sheet (core track highlighted, instructions at key points) • audited V3 test (total mark = 20) • timing audit (`audit_difficulté_parcours.md`) • Year 10 teaching model (formats A/B, section 7, homework policy Part 9) • Homework Charter (root of the folder, handed out and signed in S0 of Ch. 00) • chapter “**Gammes du soir**” (evening drills) sheet (`GammesDuSoir.pdf`, photocopied for S0) • Chapter work plan.

Safeguard never to lose sight of: “no technical virtuosity in calculation is expected”. This guide stages traps and builds proofs: it never lengthens expressions just to “make it hard”.